Holomorphic Functions of Slow Growth on Nested Covering Spaces of Compact Manifolds

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Abstract. Let $Y$ be an infinite covering space of a projective manifold $M$ in $\mathbb{P}^N$ of dimension $n \geq 2$. Let $C$ be the intersection with $M$ of at most $n-1$ generic hypersurfaces of degree $d$ in $\mathbb{P}^N$. The preimage $X$ of $C$ in $Y$ is a connected submanifold. Let $\phi$ be the smoothed distance from a fixed point in $Y$ in a metric pulled up from $M$. Let $\mathcal{O}_\phi(X)$ be the Hilbert space of holomorphic functions $f$ on $X$ such that $f^2 e^{-\phi}$ is integrable on $X$, and define $\mathcal{O}_\phi(Y)$ similarly. Our main result is that (under more general hypotheses than described here) the restriction $\mathcal{O}_\phi(Y) \to \mathcal{O}_\phi(X)$ is an isomorphism for $d$ large enough.

This yields new examples of Riemann surfaces and domains of holomorphy in $\mathbb{C}^n$ with corona. We consider the important special case when $Y$ is the unit ball $B$ in $\mathbb{C}^n$, and show that for $d$ large enough, every bounded holomorphic function on $X$ extends to a unique function in the intersection of all the nontrivial weighted Bergman spaces on $B$. Finally, assuming that the covering group is arithmetic, we establish three dichotomies concerning the extension of bounded holomorphic and harmonic functions from $X$ to $B$. 

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