Symplectic Geometry of the Moduli Space of Flat Connections on a Riemann Surface: Inductive Decompositions and Vanishing Theorems

Lisa C. Jeffrey and Jonathan Weitsman

Abstract. This paper treats the moduli space $M_{g,1}(\Lambda)$ of representations of the fundamental group of a Riemann surface of genus $g$ with one boundary component which send the loop around the boundary to an element conjugate to $\exp \Lambda$, where $\Lambda$ is in the fundamental alcove of a Lie algebra. We construct natural line bundles over $M_{g,1}(\Lambda)$ and exhibit natural homology cycles representing the Poincaré dual of the first Chern class. We use these cycles to prove differential equations satisfied by the symplectic volumes of these spaces. Finally we give a bound on the degree of a nonvanishing element of a particular subring of the cohomology of the moduli space of stable bundles of coprime rank $k$ and degree $d$.

\footnote{In this paper the term ‘volume’ refers to the symplectic volume, or more generally to the integral of the top exterior power of a presymplectic form.}

Received by the editors June 3, 1999; revised November 1, 1999.

This material is based on work supported by the National Science Foundation under Grant No. DMS-9306029, and by grants from NSERC, FCAR, the Alfred P. Sloan Foundation and the Harmon Duncombe Foundation. Supported in part by NSF grant DMS/94-03567, and by NSF Young Investigator grant DMS/94-57821.

AMS subject classification: 58F05.