The Level 2 and 3 Modular Invariants for the Orthogonal Algebras

Terry Gannon

Abstract. The ‘1-loop partition function’ of a rational conformal field theory is a sesquilinear combination of characters, invariant under a natural action of $\text{SL}_2(\mathbb{Z})$, and obeying an integrality condition. Classifying these is a clearly defined mathematical problem, and at least for the affine Kac-Moody algebras tends to have interesting solutions. This paper finds for each affine algebra $B_r^{(1)}$ and $D_r^{(1)}$ all of these at level $k \leq 3$. Previously, only those at level 1 were classified. An extraordinary number of exceptionals appear at level 2—the $B_r^{(1)}, D_r^{(1)}$ level 2 classification is easily the most anomalous one known and this uniqueness is the primary motivation for this paper. The only level 3 exceptionals occur for $B_2^{(1)} \cong C_2^{(1)}$ and $D_2^{(1)}$. The $B_{2,3}$ and $D_{2,3}$ exceptionals are cousins of the $E_6$-exceptional and $E_8$-exceptional, respectively, in the A-D-E classification for $A_1^{(1)}$, while the level 2 exceptionals are related to the lattice invariants of affine $u(1)$.