On Some $q$-Analogs of a Theorem of Kostant-Rallis

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Abstract. In the first part of this paper generalizations of Hesselink's $q$-analog of Kostant's multiplicity formula for the action of a semisimple Lie group on the polynomials on its Lie algebra are given in the context of the Kostant-Rallis theorem. They correspond to the cases of real semisimple Lie groups with one conjugacy class of Cartan subgroup. In the second part of the paper a $q$-analog of the Kostant-Rallis theorem is given for the real group $\text{SL}(4, \mathbb{R})$ (that is $\text{SO}(4)$ acting on symmetric $4 \times 4$ matrices). This example plays two roles. First it contrasts with the examples of the first part. Second it has implications to the study of entanglement of mixed 2 qubit states in quantum computation.