Spectral Problems for Non-Linear Sturm-Liouville Equations with Eigenparameter Dependent Boundary Conditions

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Abstract. The nonlinear Sturm-Liouville equation

\[ -(py')' + qy = \lambda (1 - f)ry \] on \([0, 1]\]

is considered subject to the boundary conditions

\[ (a_j \lambda + b_j)y(j) = (c_j \lambda + d_j)(py')'(j), \quad j = 0, 1. \]

Here \(a_0 = 0 = c_0\) and \(p, r > 0\) and \(q\) are functions depending on the independent variable \(x\) alone, while \(f\) depends on \(x, y\) and \(y'\). Results are given on existence and location of sets of \((\lambda, y)\) bifurcating from the linearized eigenvalues, and for which \(y\) has prescribed oscillation count, and on completeness of the \(y\) in an appropriate sense.