Localization in Categories of Complexes and Unbounded Resolutions

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Abstract. In this paper we show that for a Grothendieck category $\mathcal{A}$ and a complex $E$ in $\mathbf{C}(\mathcal{A})$ there is an associated localization endofunctor $\ell$ in $\mathbf{D}(\mathcal{A})$. This means that $\ell$ is idempotent (in a natural way) and that the objects that go to 0 by $\ell$ are those of the smallest localizing (= triangulated and stable for coproducts) subcategory of $\mathbf{D}(\mathcal{A})$ that contains $E$. As applications, we construct $K$-injective resolutions for complexes of objects of $\mathcal{A}$ and derive Brown representability for $\mathbf{D}(\mathcal{A})$ from the known result for $\mathbf{D}(R\text{-mod})$, where $R$ is a ring with unit.