Spectral Estimates for Towers of Noncompact Quotients

Anton Deitmar and Werner Hoffman

Abstract. We prove a uniform upper estimate on the number of cuspidal eigenvalues of the $\Gamma$-automorphic Laplacian below a given bound when $\Gamma$ varies in a family of congruence subgroups of a given reductive linear algebraic group. Each $\Gamma$ in the family is assumed to contain a principal congruence subgroup whose index in $\Gamma$ does not exceed a fixed number. The bound we prove depends linearly on the covolume of $\Gamma$ and is deduced from the analogous result about the cut-off Laplacian. The proof generalizes the heat-kernel method which has been applied by Donnelly in the case of a fixed lattice $\Gamma$. 