Abstract. A classical theorem of Hermite and Joubert asserts that any field extension of degree $n = 5$ or $6$ is generated by an element whose minimal polynomial is of the form $\lambda^n + c_1 \lambda^{n-1} + \cdots + c_{n-1} \lambda + c_n$ with $c_1 = c_3 = 0$. We show that this theorem fails for $n = 3^m$ or $3^m + 3^l$ (and more generally, for $n = p^m$ or $p^m + p^l$, if $3$ is replaced by another prime $p$), where $m > l \geq 0$. We also prove a similar result for division algebras and use it to study the structure of the universal division algebra $UD(n)$.

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