THE BEHAVIOUR OF LEGENDRE AND ULTRASPHERICAL POLYNOMIALS IN $L_p$-SPACES

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ABSTRACT. We consider the analogue of the $A(p)-$problem for subsets of the Legendre polynomials or more general ultraspherical polynomials. We obtain the “best possible” result that if $2 < p < 4$ then a random subset of $N$ Legendre polynomials of size $N^{1/p-1}$ spans an Hilbertian subspace. We also answer a question of König concerning the structure of the space of polynomials of degree $n$ in various weighted $L_p$-spaces.

Received by the editors March 31, 1998; revised July 6, 1998.

The first author was supported by NSF grant DMS-9500125. The second author was partially supported by the Landau Center for Research in Mathematical Analysis and Related Areas, sponsored by the Minerva Foundation (Germany).

AMS subject classification: 42C10, 33C45, 46B07.