SMOOTH FORMAL EMBEDDINGS
AND THE RESIDUE COMPLEX

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ABSTRACT. Let $\pi : X \to S$ be a finite type morphism of noetherian schemes. A smooth formal embedding of $X$ (over $S$) is a bijective closed immersion $X \subset \mathfrak{x}$, where $\mathfrak{x}$ is a noetherian formal scheme, formally smooth over $S$. An example of such an embedding is the formal completion $\mathfrak{x} = Y/S$ where $X \subset Y$ is an algebraic embedding. Smooth formal embeddings can be used to calculate algebraic De Rham (co)homology.

Our main application is an explicit construction of the Grothendieck residue complex when $S$ is a regular scheme. By definition the residue complex is the Cousin complex of $\pi^! O_S$, as in [RD]. We start with I-C. Huang's theory of pseudofunctors on modules with 0-dimensional support, which provides a graded sheaf $\bigoplus q K^q X \to S$. We then use smooth formal embeddings to obtain the coboundary operator $\partial : K^q X \to K^{q+1} X \to S$. We exhibit a canonical isomorphism between the complex $(K^q X \to S, \partial)$ and the residue complex of [RD]. When $\pi$ is equidimensional of dimension $n$ and generically smooth we show that $H^n K^q X \to S$ is canonically isomorphic to the sheaf of regular differentials of Kunz-Waldi [KW].

Another issue we discuss is Grothendieck Duality on a noetherian formal scheme $\mathfrak{x}$. Our results on duality are used in the construction of $K^q X \to S$.