SOME NORMS ON UNIVERSAL ENVELOPING ALGEBRAS

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ABSTRACT. The universal enveloping algebra, \( U(\mathfrak{g}) \), of a Lie algebra \( \mathfrak{g} \) supports some norms and seminorms that have arisen naturally in the context of heat kernel analysis on Lie groups. These norms and seminorms are investigated here from an algebraic viewpoint. It is shown that the norms corresponding to heat kernels on the associated Lie groups decompose as product norms under the natural isomorphism
\[
U(\mathfrak{g}_1 \oplus \mathfrak{g}_2) \cong U(\mathfrak{g}_1) \otimes U(\mathfrak{g}_2).
\]
The seminorms corresponding to Green’s functions are examined at a purely Lie algebra level for \( \text{sl}(2, \mathbb{C}) \). It is also shown that the algebraic dual space \( U' \) is spanned by its finite rank elements if and only if \( \mathfrak{g} \) is nilpotent.

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