ASYMPTOTIC SHAPE OF FINITE PACKINGS

KÁROLY BÖRÖCZY, JR., AND UWE SCHNELL

ABSTRACT. Let $K$ be a convex body in $\mathbb{E}^d$ and denote by $C_n$ the set of centroids of $n$ non-overlapping translates of $K$. For $g > 0$, assume that the parallel body $\text{conv} C_n + gK$ of $\text{conv} C_n$ has minimal volume. The notion of parametric density (see [21]) provides a bridge between finite and infinite packings (see [4] or [14]). It is known that there exists a maximal $g_0(K) \geq \frac{1}{32d^2}$ such that $\text{conv} C_n$ is a segment for $g < g_0$ (see [5]). We prove the existence of a minimal $g_c(K) \leq d + 1$ such that if $g > g_c$ and $n$ is large then the shape of $\text{conv} C_n$ can not be too far from the shape of $K$. For $d = 2$, we verify that $g_s = g_c$. For $d \geq 3$, we present the first example of a convex body with known $g_s$ and $g_c$; namely, we have $g_s = g_c = 1$ for the parallelotope.