FACTORIZATION IN THE INVERTIBLE GROUP OF A $C^*$-ALGEBRA

MICHAEL J. LEEN

Abstract. In this paper we consider the following problem: Given a unital $C^*$-algebra $A$ and a collection of elements $S$ in the identity component of the invertible group of $A$, denoted $\text{inv}_0(A)$, characterize the group of finite products of elements of $S$. The particular $C^*$-algebras studied in this paper are either unital purely infinite simple or of the form $(A \otimes K)^+$, where $A$ is any $C^*$-algebra and $K$ is the compact operators on an infinite dimensional separable Hilbert space. The types of elements used in the factorizations are unipotents ($1+$ nilpotent), positive invertibles and symmetries ($s^2 = 1$). First we determine the groups of finite products for each collection of elements in $(A \otimes K)^+$. Then we give upper bounds on the number of factors needed in these cases. The main result, which uses results for $(A \otimes K)^+$, is that for $A$ unital purely infinite and simple, $\text{inv}_0(A)$ is generated by each of these collections of elements.

Received by the editors November 16, 1996.
AMS subject classification: 46L05.