HOMOMORPHISMS FROM $C(X)$ INTO $C^*$-ALGEBRAS

HUAXIN LIN

ABSTRACT. Let $A$ be a simple $C^*$-algebra with real rank zero, stable rank one and weakly unperforated $K_0(A)$ of countable rank. We show that a monomorphism $\phi: C(S^2) \to A$ can be approximated pointwise by homomorphisms from $C(S^2)$ into $A$ with finite dimensional range if and only if certain index vanishes. In particular, we show that every homomorphism $\phi$ from $C(S^2)$ into a UHF-algebra can be approximated pointwise by homomorphisms from $C(S^2)$ into the UHF-algebra with finite dimensional range. As an application, we show that if $A$ is a simple $C^*$-algebra of real rank zero and is an inductive limit of matrices over $C(S^2)$ then $A$ is an AF-algebra. Similar results for tori are also obtained. Classification of $Hom(C(X), A)$ for lower dimensional spaces is also studied.

Received by the editors May 18, 1995.
AMS subject classification: 46L05, 46L80, 46L35.
Key words and phrases: Homomorphism of $C(S^2)$, approximation, real rank zero, classification