CLASSIFYING PL 5-MANIFOLDS BY REGULAR GENUS: THE BOUNDARY CASE

MARIA RITA CASALI

ABSTRACT. In the present paper, we face the problem of classifying classes of orientable PL 5-manifolds $M^5$ with $h \geq 1$ boundary components, by making use of a combinatorial invariant called regular genus $G(M^5)$. In particular, a complete classification up to regular genus five is obtained:

$$G(M^5) = 0 \implies M^5 \cong \#_{i=1}^h (\mathbb{S}^4 \times \mathbb{S}^1)^{h_i} \boxtimes Y_i,$$

where $\mathcal{B} = G(\partial M^5)$ denotes the regular genus of the boundary $\partial M^5$ and $\boxtimes Y_i$ denotes the connected sum of $h \geq 1$ orientable 5-dimensional handlebodies $Y_i$ of genus $g_i \geq 0$ ($i = 1, \ldots, h$), so that $\sum_{i=1}^h \alpha_i = \mathcal{B}$.

Moreover, we give the following characterizations of orientable PL 5-manifolds $M^5$ with boundary satisfying particular conditions related to the “gap” between $G(M^5)$ and either $G(\partial M^5)$ or the rank of their fundamental group $\text{rk}(\pi_1(M^5))$:

$$G(\partial M^5) = G(M^5) = 0 \iff M^5 \cong (\mathcal{B}) \boxtimes Y_i \iff$$

$$G(\partial M^5) = \mathcal{B} = G(M^5) - 1 \iff M^5 \cong (\mathbb{S}^4 \times \mathbb{S}^1)^{h_i} \boxtimes Y_i \iff$$

$$G(\partial M^5) = \mathcal{B} = G(M^5) - 2 \iff M^5 \cong \#_{i=1}^h (\mathbb{S}^4 \times \mathbb{S}^1)^{h_i} \boxtimes Y_i \iff$$

$$G(M^5) = \text{rk}(\pi_1(M^5)) = 0 \iff M^5 \cong \#_{i=1}^h (\mathbb{S}^4 \times \mathbb{S}^1)^{h_i} \boxtimes Y_i.$$

Further, the paper explains how the above results (together with other known properties of regular genus of PL manifolds) may lead to a combinatorial approach to 3-dimensional Poincaré Conjecture.

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