SWEEPING OUT PROPERTIES
OF OPERATOR SEQUENCES

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ABSTRACT. Let $L_p = L_p(X, \mu)$, $1 \leq p \leq \infty$, be the usual Banach Spaces of real valued functions on a complete non-atomic probability space. Let $(T_1, \ldots, T_K)$ be $L_2$-contractions. Let $0 < \varepsilon < \delta \leq 1$. Call a function $f$ a $\delta$-spanning function if $\|f\|_2 = 1$ and if $\|T_k f - Q_k f\|_2 \geq \delta$ for each $k = 1, \ldots, K$, where $Q_0 = 0$ and $Q_k$ is the orthogonal projection on the subspace spanned by $(T_1 f, \ldots, T_k f)$. Call a function $h$ a $(\delta, \varepsilon)$-sweeping function if $\|h\|_\infty \leq 1$, $\|h\|_1 < \varepsilon$, and if $\max_{1 \leq k \leq K} |T_k h| > \delta - \varepsilon$ on a set of measure greater than $1 - \varepsilon$. The following is the main technical result, which is obtained by elementary estimates. There is an integer $K = K(\varepsilon, \delta) \geq 1$ such that if $f$ is a $\delta$-spanning function, and if the joint distribution of $(f, T_1 f, \ldots, T_K f)$ is normal, then $h = \left( \left\{ f \wedge M \vee (-M) \right\} \right) / M$ is a $(\delta, \varepsilon)$-sweeping function, for some $M > 0$. Furthermore, if $T_k$s are the averages of operators induced by the iterates of a measure preserving ergodic transformation, then a similar result is true without requiring that the joint distribution is normal. This gives the following theorem on a sequence $(T_i)$ of these averages. Assume that for each $K \geq 1$ there is a subsequence $(T_{i_1}, \ldots, T_{i_K})$ of length $K$, and a $\delta$-spanning function $f_K$ for this subsequence. Then for each $\varepsilon > 0$ there is a function $h$, $0 \leq h \leq 1$, $\|h\|_1 < \varepsilon$, such that $\lim \sup \left| T_i h \right| \geq \delta$ a.e. Another application of the main result gives a refinement of a part of Bourgain’s “Entropy Theorem”, resulting in a different, self contained proof of that theorem.

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