Recent Advances in Arithmetic and Hyperbolic Geometry
Avancées récentes en géométrie arithmétique et hyperbolique

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NILS BRUIN, Simon Fraser University
Prym varieties in genus 4

Many arithmetic properties of hyperbolic curves become apparent from embeddings into abelian varieties, in particular their Jacobians. For special curves, particularly those that arise as unramified double covers of another curve (of genus g), the Jacobian variety itself is decomposable. This leads to Prym varieties. These are principally polarized abelian varieties of dimension g-1. Having an explicit description of these varieties is an essential ingredient in many computational methods. We discuss an explicit construction for g equal to 4. This is joint work with Emre Can Sertoz.

XI CHEN, University of Alberta
$\mathbb{A}^1$ *curves on Log K3 Surfaces*

It is known that there are infinitely many rational curves on almost every projective K3 surface. It is expected that the same holds on every log K3 surface, with rational curves replaced by $\mathbb{A}^1$ curves. However, there are log K3 surfaces which do not contain any $\mathbb{A}^1$ curves. Together with Yi Zhu, we classified all genuine log K3 surfaces that contain $\mathbb{A}^1$ curves and proved that these log surfaces contain infinitely many $\mathbb{A}^1$ curves.

DRAGOS GHIOCA, University of British Columbia
Unlikely intersections in arithmetic dynamics

Motivated by the classical conjectures formulated independently by Bombieri-Masser-Zannier, and also by Pink and Zilber regarding intersections of subvarietes X of semiabelian varieties G with unions of algebraic subgroups of G of codimension larger than the dimension of X, we formulate the following dynamical analogue. In our setting, the ambient space G is the affine space $\mathbb{A}^N$ endowed with the coordinatewise action Φ of N one-variable polynomials $f_1, \dots, f_N$ of degree larger than 1, which are not conjugated to monomials or to $\pm$Chebyshev polynomials. Then for a subvariety X of G of dimension d, it is expected that its intersection with the union of all irreducible subvarieties of G of codimension larger than d, which are periodic under the action of Φ would not be Zariski dense in X, unless X is contained in a proper, irreducible periodic subvariety of G. In a joint work with Khoa Nguyen, we proved this conjecture when X is either a curve, or it has codimension at most equal to 2.

GORDON HEIER, University of Houston
A generalized Schmidt subspace theorem for closed subschemes

In this talk, we will present a generalized version of Schmidt's subspace theorem for closed subschemes in general position in terms of suitably defined Seshadri constants with respect to a fixed ample divisor. Our proof builds on previous work by Evertse and Ferretti, Corvaja and Zannier, and others, and uses standard techniques from algebraic geometry such as notions of positivity, blowing-ups and direct image sheaves. As an application, we recover a higher-dimensional Diophantine approximation theorem of K. F. Roth-type due to D. McKinnon and M. Roth with a significantly shortened proof, while simultaneously extending the scope of the use of Seshadri constants in this context in a natural way. Time permitting, we will also discuss applications to the degeneracy of integral points on varieties in the complement of divisors. This is joint work with Aaron Levin.

AARON LEVIN, Michigan State University
Greatest common divisors in Diophantine approximation and Nevanlinna theory

In 2003, Bugeaud, Corvaja, and Zannier gave an (essentially sharp) upper bound for the greatest common divisor $\gcd(a^n - 1, b^n - 1)$, where a and b are fixed integers and n varies over the positive integers. In contrast to the elementary statement of their result, the proof required deep results from Diophantine approximation. I will discuss a higher-dimensional generalization of their result and some related results and problems.

DAVID MCKINNON, University of Waterloo
The arithmetic puncturing problem

Let V be a projective variety over a number field, and let Z be a subset of V of codimension at least two. In 2001, Hassett and Tschinkel asked whether or not it was true that the rational points of V are potentially Zariski dense if and only if the Z-integral points of V are potentially Zariski dense. The answer to this question is still mysteriously unknown, but there have been a few tantalizing bits of progress, some of which I may even have time to discuss in my talk.

MARC-HUBERT NICOLE, Université d'Aix-Marseille (AMU)
Families of Modular Forms on Drinfeld Modular Varieties for GL_N

Classical modular curves associated to GL_2 are moduli spaces of elliptic curves with additional structure. Taking advantage of the analogy between number fields and function fields, Drinfeld modules (of rank 2) were introduced as a good analogue of elliptic curves. While there are no Shimura varieties associated to the general linear group GL_N for $N > 2$, the situation is sharply different over function fields. The Drinfeld modular variety for GL_N is the moduli space of Drinfeld modules of rank N (with level structure). It is a smooth, affine scheme of dimension N-1. In this talk, I will explain how analogues of well-established theories in the classical context extend to Drinfeld modular varieties and their modular forms: notably Hida's algebraic theory of families of modular forms of slope zero, but also a continuous analogue of Coleman's analytic theory for modular forms of finite slope. Joint work with G. Rosso (Concordia Univ./Cambridge Univ.).

JUNJIRO NOGUCHI, The University of Tokyo
A big Picard theorem and the Manin-Mumford conjecture

In 1981 the present speaker proved the following theorem as a generalization of Big Picard's Theorem: *If $f : \Delta^* \to A$ is a holomorphic curve from a punctured disk Δ^* into a semi-abelian variety A with an essential singularity at the puncture, then the Zariski closure of $f(\Delta^*)$ has a positive dimensional stabilizer group B and the composite $q_B \circ f : \Delta^* \to A/B$ with the natural morphism $q_B : A \to A/B$ has at most a pole at the puncture.*

In arithmetic geometry, 1983, M. Raynaud proved the Manin-Mumford conjecture stated as: *Let $X \subset A_0$ be a subvariety of an abelian variety A_0 defined over a number field. Then the Zariski closure of the set of all torsion points on X consists of finitely many translates of algebraic subgroups of A.* There are a number of generalizations and different proofs of this celebrated Theorem of Raynaud by M. Hindry, E. Hrushovski, McQuillan, ..., Pila-Zannier.

In this talk we will discuss how the above two statements are related and that the first is applied to the proof of the second through "o-minimal structure".

The present result might be a first instance of a *direct connection at the proof level* between the value distribution theory and the arithmetic (Diophantine) theory over number fields, while there have been many *analogies* between them.

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ERWAN ROUSSEAU, Aix-Marseille université
Orbifold hyperbolicity

I will present a generalization of (Kobayashi) hyperbolicity and related techniques to the setting of orbifold pairs (in the sense of Campana) which gives new perspectives on the distribution of entire curves and rational points in projective varieties. (Joint work with F. Campana and Lionel Darondeau).

MIN RU, University of Houston
Holomorphic curves into projective varieties intersecting general divisors

We establish a general Second Main Theorem type result (as well as Schmidt's subspace type theorem in Diophantine approximation) for holomorphic curves into the projective variety X intersecting general divisor D, in terms of the (birational) Nevanlinna constant $Nev_{bir}(D)$. By computing $Nev_{bir}(D)$ using the filtrations, it recovers (almost all) previous known results in this direction, as well as derive some new results for divisors which are not necessarily linear equivalent on X. The notion $Nev_{bir}(D)$ is originally defined in terms of Weil functions for use in applications, and it is proved later that it can be defined in terms of local effectivity of Cartier divisors after taking a proper birational lifting. This is a joint work with Paul Vojta.

BEHROUZ TAJI, University of Notre Dame
Hyperbolicity properties of base space of famillies of projective manifolds

Base spaces of smooth families of projective manifolds with maximal variation whose canonical bundle is pseudo-effective exhibit various hyperbolicity properties. For example, it is known that the Moduli stack of polarized varieties of general type is Brody hyperbolic (due to Popa-Taji-Wu and Viehweg-Zuo). In the sense of birational geometry this can be reformulated by saying that the base spaces of such families is of log-general type (due to Campana-Paun, Popa-Schnell and Viehweg-Zuo). I will discuss generalization of these properties to the case where variation in the birational structure of such families is not maximal.

AMOS TURCHET, University of Washington
Arithmetic and algebraic hyperbolicity of surface pairs with almost ample cotangent bundle

We will discuss arithmetic and geometric hyperbolicity properties of pairs (X, D) where X is a surface defined over a number field κ and D is a normal crossing divisor. In particular we will show that when the log cotangent sheaf $\Omega^1_X(\log D)$ is almost ample all subvarieties of X are of log general type, even when (X, D) is mildly singular. Moreover if the log cotangent sheaf is in addition globally generated $X \setminus D$ has only finitely many integral points, thus generalizing a theorem of Moriwaki. This is joint work with Kenny Ascher and Kristin DeVleming.

JORG WINKELMANN, Ruhr Universität Bochum
Tame Discrete Sets

Let X be a complex manifold. We call a discrete subset $D \subset X$ to be tame if, by applying a suitable automorphism of X, the counting function $N_r(D)$ can be made as small as desired.

This generalized the notion of "tameness" for subsets of $\mathbf{C}^n$ which has been introduced by Rosay and Rudin.

For complex linear algebraic groups with trivial character group we can prove rather strong results on tame discrete sets which parallel those obtained by Rosay and Rudin for the case of a complex vector space.