Algebraic Combinatorics
Combinatoire algébrique
(Org: **Christophe Hohlweg** and/et **Franco Saliola** (UQAM))

DREW ARMSTRONG, University of Miami
Rational Catalan Combinatorics

In this talk I will define the Catalan number $\mathrm{Cat}(x)$ corresponding to a rational number $x \in \mathbb{Q}$ outside the interval $[-1, 0]$. It satisfies the symmetry $\mathrm{Cat}(x) = \mathrm{Cat}(-x-1)$. Then I will define the "derived" Catalan number

$$\mathrm{Cat}'(x) := \mathrm{Cat}(1/(x-1)) = \mathrm{Cat}(x/(1-x)).$$

(This is a categorification of the Euclidean algorithm.) It satisfies the symmetry $\mathrm{Cat}'(x) = \mathrm{Cat}'(1/x)$. I will make the bold assertion that every nice class of Catalan objects has a generalization counted by $\mathrm{Cat}(x)$. I will provide evidence, in the form of lattice paths, noncrossing partitions and associahedra. In the case of associahedra, the symmetry $\mathrm{Cat}'(x) = \mathrm{Cat}'(1/x)$ is a topological statement about Alexander duality.

CAROLINA BENEDETTI, York University
Schubert polynomials and k-Schur functions

In this talk we study operators associated to the graph on dual k-Schur functions given by the affine grassmannian order. These operators are analog to the ones given in the r-Bruhat order by Bergeron-Sottile. This allows us to understand combinatorially the multiplication of a Schubert polynomial by a Schur function from the multiplication in the space of dual k-Schur functions. (Joint work with Nantel Bergeron).

CHRIS BERG, UQAM, LaCIM
Strong Schur functions and down operators for the affine nilCoxeter algebra.

I will introduce the concepts of strong Schur functions, first defined by Lam, Lapointe, Morse and Shimozono. Taking the combinatorial concepts of their definition, I will introduce a family of operators on the nilCoxeter algebra which were used in the proofs of several conjectures on strong Schur functions. This is all joint work with Franco Saliola and Luis Serrano.

NANTEL BERGERON, York University
Hopf monoid for supercharacter and NCSym

I am combining recent work with Thiem and with Aguiar-Thiem.

With Nat Thiem, we have defined a new basis of symmetric functions in noncommutative variables that depends on a parameter q. This new basis is natural in the context of (coarser) supercharacter theory of the unipotent uppertriangular matrices over a finite fields F_q. I will introduce this in the setting of Hopf monoid and show how to compute antipode and primitives as done with Aguiar and Thiem.

CHRISTOPHE REUTENAUER, Université du Québec à Montréal
Constructing bases of finite index subgroups of free groups using Sturmian sequences

Joint work with Jean Berstel, Clelia De Felice, Dominique Perrin, Giuseppina Rindone. The Schützenberger theory of bifix codes is extended to subsets of F, the set of factors of a Sturmian (or epiSturmian) sequence. It is shown that such a code, if maximal, is the basis of a subgroup of the free group, of index equal to the degree d of the code; d is the number of partial decodings of long words. This result extends considerably the classical fact (Morse-Hedlund) that the number of factors of length d of any Sturmian sequence is $d+1$.

ED RICHMOND, UBC

Coxeter groups, palindromic Poincaré polynomials and triangle group avoidance.

Let W be a Coxeter group. For any $w \in W$, let P_w denote its Poincaré polynomial (i.e. the generating function of the principle order ideal of w with respect to length). If W is the Weyl group of some Kac-Moody group G, then P_w is the usual Poincaré polynomial of the corresponding Schubert variety X_w.

In this talk, I will discuss joint work with W. Slofstra on detecting when the sequence of coefficients of a Poincaré polynomial are the same read forwards and backwards (i.e. palindromic). The polynomial P_w satisfies this property precisely when the Schubert variety X_w is rationally smooth. It turns out that this property is easy to detect when the Coxeter group W avoids certain rank 3 parabolic subgroups (triangle groups). One consequence is that, for many Coxeter groups, the number of elements with palindromic Poincaré polynomials is finite. Explicit enumerations and descriptions of these elements are given in special cases.

VIVIEN RIPOLL, UQAM

Limit points of root systems of infinite Coxeter groups

Let W be an infinite Coxeter group, and consider the root system constructed from its geometric representation. We study the set E of limit points of the directions of roots. As motivational examples, we describe with pictures in rank 2, 3, 4, the fractal shape of this limit set E. We define a natural geometric action of W on E and explain its properties (transitivity, orbit of a point). We also study the extreme points of the convex hull of E and its relation with the "imaginary cone" of W. (joint works with M. Dyer, Ch. Hohlweg and J.-P. Labbé, arXiv:1112.5415)

HUGH THOMAS, University of New Brunswick

Lexicographically first subwords in Coxeter groups and quiver representations

Fix a reduced word for the longest element w_0 of a finite Coxeter group W, and then, for each $w \in W$, find the lexicographically first reduced subword for w in the fixed word for w_0. Sorting order, introduced by Drew Armstrong in 2007, is inclusion order on these subwords. Lexicographically first reduced subwords also play an important role in total positivity, where they go by the name of "positive distinguished subexpressions": leftmost reduced words in a maximal Grassmannian permutation naturally index cells in the totally positive part of the corresponding Grassmannian. It turns out that for certain reduced words for w_0 (those which are c-sorting words), including those relevant for total positivity of Grassmannians, the lexicographically first subwords can be described using quiver representations. I will explain this (without assuming prior knowledge of representation theory of quivers). This talk will be based on joint work with Steffen Oppermann and Idun Reiten, arXiv:1205.3268.

STEPHANIE VAN WILLIGENBURG, UBC

Maximal supports and Schur-positivity among connected skew shapes

The Schur-positivity order on skew shapes is denoted by $B < A$ if the difference of their respective Schur functions is a positive linear combination of Schur functions. It is an open problem to determine those connected skew shapes that are maximal with respect to this ordering. In this talk we see that to determine the maximal connected skew shapes in the Schur-positivity order it is enough to consider a special class of ribbon shapes. We also explicitly determine the support for these ribbon shapes.

This is joint work with Peter McNamara.

MIKE ZABROCKI, York University

Another Schur-like basis in the algebra of Non-Commutative Symmetric Functions

The algebra of non-commuative symmetric functions (NSym) is the free algebra generated by one element of each degree and it is the dual Hopf algebra of the quasi-symmetric functions (QSym) (see for example: "Noncommutative symmetric functions" by I. Gelfand, et. al. and "Multipartite P-partitions and inner products of skew Schur functions" by I. Gessel).

Since this algebra was first studied, it was believed that 'the' analogue of the Schur symmetric functions in NSym must be the ribbon basis. Recent results however have called that into question since the dual to the Quasi-Schur functions by J. Haglund et. al. is 'Schur-like' and its commutative image in the ring of symmetric functions is a Schur function. In this talk I will present another candidate for a 'Schur-like' basis that is based on a determinantal formula and show that it has many interesting properties including a (right) Pieri and Littlewood-Richardson rule, a formula using creation operators, a projection onto the Schur symmetric functions, a Murnaghan-Nakayama rule, a generalization to Hall-Littlewood symmetric functions, etc. These 'Schur-like' bases make us rethink what might be possible in NSym and QSym. This is joint work with Chris Berg, Nantel Bergeron, Franco Saliola and Luis Serrano.