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Rational Catalan Combinatorics

In this talk I will define the Catalan number $\mathrm{Cat}(x)$ corresponding to a rational number $x \in \mathbb{Q}$ outside the interval $[-1, 0]$. It satisfies the symmetry $\mathrm{Cat}(x) = \mathrm{Cat}(-x-1)$. Then I will define the "derived" Catalan number

$$\mathrm{Cat}'(x) := \mathrm{Cat}(1/(x-1)) = \mathrm{Cat}(x/(1-x)).$$

(This is a categorification of the Euclidean algorithm.) It satisfies the symmetry $\mathrm{Cat}'(x) = \mathrm{Cat}'(1/x)$. I will make the bold assertion that every nice class of Catalan objects has a generalization counted by $\mathrm{Cat}(x)$. I will provide evidence, in the form of lattice paths, noncrossing partitions and associahedra. In the case of associahedra, the symmetry $\mathrm{Cat}'(x) = \mathrm{Cat}'(1/x)$ is a topological statement about Alexander duality.