A new small value estimate on $\mathbb{C} \times \mathbb{C}^*$

Typical constructions of auxiliary functions from transcendental number theory yield polynomials with integers coefficients taking small values at many points of a finitely generated subgroup of an algebraic group. For future progress in Algebraic independence, it is desirable to study the cases where these values are not small enough so that we can apply Philippon’s criterion of algebraic independence. In this talk, we present such a new situation where we achieve an almost optimal result. More precisely, we assume the existence of a sequence of polynomials in $\mathbb{Z}[X_1, X_2]$ of controlled degree and height taking small values at points of the form $(\xi + ir, \eta s^i)$ ($i = 0, 1, 2, \ldots$) for fixed non-zero rational numbers $r$ and $s \neq \pm 1$ and show that this is possible if and only if both $\xi$ and $\eta$ are algebraic.