The large sieve inequality implies that if one takes the integers less than $x$, and removes around half of the residue classes modulo each prime, then the resulting set must have size $\ll \sqrt{x}$. This bound is sharp in the case where one removes the quadratic non-residues modulo each prime, in which case the set of squares is left behind. The inverse conjecture for the large sieve proposes, amongst other things, that there are no essentially different examples where this $\sqrt{x}$ bound is sharp.

In this talk I will outline how, by combining the large sieve, the larger sieve, and some basic ideas from additive combinatorics, one can prove some results in the direction of the inverse conjecture. This is joint work with Ben Green.