We present new extensions to a method for constructing several families of solvable one-dimensional time-homogeneous diffusions. Our approach is based on a dual application of the so-called diffusion canonical transformation method that combines smooth monotonic mappings and measure changes via Doob-h transforms. This gives rise to new multi-parameter solvable diffusions that are generally divided into two main classes; the first is specified by having affine (linear) drift with various resulting nonlinear diffusion coefficient functions, while the second class allows for several specifications of a (generally nonlinear) diffusion coefficient with resulting nonlinear drift function. The theory is applicable to diffusions with either singular and/or non-singular endpoints. As part of the results in this paper, we also present a complete boundary classification and martingale characterization of the newly developed diffusion families. The first class of models, having linear drift and nonlinear (state-dependent) volatility functions, is useful for equity derivative pricing in finance, while the second class of diffusions contains new models that are mean-reverting and which are applicable in areas such as interest rate modeling. As specific examples of the first class of affine drift models, we present explicit results for three new families of models. For the second class of nonlinear drift models, we give examples of solvable subfamilies of mean-reverting diffusions and derive some closed-form integral formulas for conditional expectations of functionals that can be used to price bonds and bond options.