Stochastic volatility models have been in place for some years now. A natural extension of the latter ones is a multivariate model with stochastic correlation. And indeed, the performance of a portfolio or a multi-dimensional derivative depends very much on the joint behaviour of the underlyings, i.e. the covariances, which are not constant over time. However, one of the main problems with the modelling of correlation is intractability because the number of parameters grows quite fast as dimensions increase. The model treated here is based on a stochastic principal component model, which reduces the dimension of the original problem. We reduce complexity by modelling the eigenvalues of the assets instead of the full covariance matrix. We set the eigenvectors constant but assume the eigenvalues stochastic. An empirical analysis shows that the eigenvalues are driven by two mean-reverting components, one which varies in the order of days and the other one which varies in the order of months. Our approach allows a multi-dimensional extension of the Heston model with stochastic volatility, stochastic correlation among assets, between variances and assets as well as between assets and correlation. The proposed model is applied to price end-point as well as path-dependent two-asset options. A closed-form solution for barrier options under stochastic correlation has not been found. Thus, we show how perturbation theory can be used to find easy and well converging approximations to non-vanilla options on two correlated underlyings.