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Free probability and the invariant subspace problem for von Neumann algebras

Does every operator T on a Hilbert space H have a non-trivial closed invariant subspace? This is the famous and still open “invariant subspace problem” for operators on a Hilbert space. A natural generalization of the problem is: Let M be a von Neumann algebra on a Hilbert space H . Does every operator T in M have a non-trivial closed invariant subspace K affiliated with M ? (K is affiliated with M , iff the orthogonal projection on K belongs to M .) In the special case, when M is a II_1 -factor (i.e., a infinite dimensional von Neumann factor with a bounded trace), it turns out, that “almost all” operators in M have non-trivial closed invariant subspaces affiliated with M . More precisely, it holds for all operators in M for which L. G. Brown’s spectral distribution measure for T is not concentrated in a single point of the complex plane. The result is obtained in collaboration with Hanne Schultz, and it relies in a crucial way on Voiculescu’s free probability theory.