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Interpolation on Symmetric Spaces via the Generalized Polar Decomposition

We construct interpolation operators for functions taking values in a symmetric space — a smooth manifold with an inversion symmetry about every point. Key to our construction is the observation that every symmetric space can be realized as a homogeneous space whose cosets have canonical representatives by virtue of the generalized polar decomposition — a generalization of the well-known factorization of a real nonsingular matrix into the product of a symmetric positive-definite matrix times an orthogonal matrix. By interpolating these canonical coset representatives, we derive a family of structure-preserving interpolation operators for symmetric space-valued functions. As applications, we construct interpolation operators for the space of Lorentzian metrics, the space of symmetric positive-definite matrices, and the Grassmannian. In the case of Lorentzian metrics, our interpolation operators provide a family of finite elements for numerical relativity that are frame-invariant and have signature which is guaranteed to be Lorentzian pointwise. We illustrate their potential utility by interpolating the Schwarzschild metric numerically. This is joint work with Evan Gawlik.