## Convex and Discrete Geometry, and Geometric Analysis
## Géométrie convexe et discrète, et analyse géométrique
(Org: **Alexander Litvak**, **Anna Lytova** and/et **Vladyslav Yaskin** (Alberta))

**KAROLY BEZDEK**, University of Calgary
*Contact numbers for sphere packings*

In discrete geometry, the contact number of a given finite number of non-overlapping spheres was introduced as a generalization of Newton's kissing number. This notion has not only led to interesting mathematics, but has also found applications in the science of self-assembling materials, such as colloidal matter. In this talk, we investigate the problem in general and emphasize important special cases including contact numbers of minimally rigid and totally separable sphere packings. We also discuss the complexity of recognizing contact graphs in a fixed dimension. Moreover, we list some conjectures and open problems. This is a joint work with Muhammad A. Khan (University of Calgary).

**IVAN IURCHENKO**, University of Alberta
*On affine invariant points*

Affine invariant point is a function $f$ from the set of convex bodies in $\mathbb{R}^n$ into $\mathbb{R}^n$ satisfying the condition $f(\varphi(K)) = \varphi(f(K))$ for any convex body $K$ and any affine transformation $\varphi$. We design a new class of affine invariant points. Denoting by $\mathcal{F}$ the set of all affine points we answer the question by Grünbaum how big is the set $\{f(K) \mid f \in \mathcal{F}\}$ for any given convex body $K$.

**MUHAMMAD A. KHAN**, University of Calgary
*The geometry of homothetic covering and illumination*

At a first glance, the problem of illuminating the boundary of a convex body by minimal number of external light sources and the problem of economically covering a convex body by its smaller positive homothetic copies appear to be quite different. They are in fact two sides of the same coin and give rise to one of the important longstanding open questions in discrete geometry, namely, the Illumination Conjecture – also known as the Hadwiger Conjecture or the Covering Conjecture. In this talk, we discuss recent advances towards this conjecture and some approaches to potentially make further progress. Moreover, we describe some of our new results in this direction. This is joint work with Károly Bezdek (University of Calgary).

**JAEGIL KIM**, University of Alberta
*Distribution functions of sections and projections of convex bodies*

Typically, when we are given the section (or projection) function of a convex body, it means that in each direction we know the size of the central section (or projection) perpendicular to this direction. Suppose now that we can only get the information about the sizes of sections (or projections), and not about the corresponding directions. In this talk we will discuss to what extent the distribution function of the areas of central sections (or projections) of a convex body can be used to derive some information about the body, its volume, etc. (Joint work with V. Yaskin and A. Zvavitch)

**GALYNA LIVSHYTS**, Georgia Institute of Technology
*On infinitesimal versions of Log-Brunn-Minkowski and related inequalities*

Log-Brunn-Minkowski conjecture was proposed by K.J. Boroczky, E. Lutwak, D. Yang and G. Zhang, and it suggests a strengthening of the classical Brunn-Minkowski inequality in the case of symmetric convex sets.

We determine the infinitesimal version of the log-Brunn-Minkowski inequality. As a consequence, we obtain a strong Poincare-type inequality in the case of unconditional convex sets, as well as symmetric convex sets on the plane.

Using it, we establish the validity of the log-Brunn-Minkowski inequality for any pair of symmetric convex sets which are close enough to a Euclidean ball of any radius. We also establish the validity of dimensional Brunn-Minkowski inequality for any pair of (not necessarily symmetric) convex sets near a ball, with respect to a rotation invariant log-concave measure.

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**ARNAUD MARSIGLIETTI**, IMA, University of Minnesota
*Do Minkowski averages get progressively more convex?*

Let us define, for a compact set $A \subset \mathbb{R}^n$, the Minkowski averages of $A$:

$$A(k) = \left\{ \frac{a_1 + \cdots + a_k}{k} : a_1, \ldots, a_k \in A \right\} = \frac{1}{k}\Big( \underbrace{A + \cdots + A}_{k \text{ times}} \Big).$$

Shapley, Folkmann and Starr (1969) proved that $A(k)$ converges to the convex hull of $A$ in Hausdorff distance as $k$ goes to $\infty$. Bobkov, Madiman and Wang (2011) conjectured that when one has convergence in the Shapley-Folkmann-Starr theorem in terms of a volume deficit, then this convergence is actually monotone. More precisely, they conjectured that $|A(k)|$ is non-decreasing, where $|\cdot|$ denotes Lebesgue measure.

In this talk, we show that this conjecture holds true in dimension 1 but fails in dimension $n \geq 12$. We also consider whether one can have monotonicity when measured using alternate measures of non-convexity, including the Hausdorff distance, effective standard deviation, and a non-convexity index of Schneider.

(Joint work with Matthieu Fradelizi, Mokshay Madiman and Artem Zvavitch.)

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**SERGII MYROSHNYCHENKO**, Kent State University
*On polytopes with congruent projections*

Let $P$ and $Q$ be two polytopes in $\mathbb{E}^n$, $n \geq 3$, such that their projections onto any $k$-dimensional subspace, $2 \leq k \leq n-1$, are congruent (i.e. coincide up to a rigid motion). We show that $P$ and $Q$ coincide up to translation and reflection in the origin."

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**ELIZAVETA REBROVA**, University of Michigan
*Regularization of the norm of random matrices*

We study $n \times n$ matrices $A$ with i.i.d. entries having zero mean and unit variance. If the entries are also subgaussian, then the operator norm $\|A\| \sim O(\sqrt{n})$ with high probability, but without subgaussian assumption the norm can be much larger, in some examples it is up to $O(n)$.

So, we are motivated by the question: what is it in the structure of a heavy-tailed matrix that makes its norm to blow up? We show that with high probability the problem is "local": there is a $\varepsilon n \times \varepsilon n$ sub-matrix $A_0$ (for any $\varepsilon > 0$, i.e. as small as we want), deletion of which regularizes the norm

$$\|A \setminus A_0\| \leq C(\varepsilon)\sqrt{n}$$

We will also discuss the dependence of the norm constant $C(\varepsilon)$ on size parameter $\varepsilon$ (we have it optimal up to a logarithmic factor) and how second moment condition is crucial for any "local" regularization. This is a joint work with Roman Vershynin.

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**SAMUEL REID**, Calgary

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**MATTHEW STEPHEN**, University of Alberta
*On convex intersection bodies and unique determination problems for convex bodies*

We describe a general result which ensures counter-examples for certain problems of unique determination for convex bodies. Using this result, we show a convex body $K \subset \mathbb{R}^n$ is not uniquely determined by its convex intersection body $CI(K)$, introduced by Meyer and Reisner in 2011.

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**KATERYNA TATARKO**, University of Alberta
*On the mean curvature vector of a special class of submanifolds in a Riemannian submersion*

We examine the extrinsic geometry of submanifolds which are generated by the restriction of a Riemannian submersion to submanifolds in the base. In particular, we give a representation of the mean curvature vector of submanifolds of the restriction type in the Riemannian submersion space. Joint work with A. Yampolsky.

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**KONSTANTIN TIKHOMIROV**, University of Alberta
*Randomized coverings by homothetic copies, and illumination*

We consider certain models of illuminating convex bodies by random independent light sources. This is a joint work with Galyna Livshyts.

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**SHARDUL VIKRAM**, Concordia University
*A New Approach to the Planar Fractional Minkowski problem via a Curvature Flow*

We will present a planar anisotropic curvature flow on the space of smooth, symmetric and strictly convex bodies of the Euclidean plane. We study its long term existence and show that the solutions of the flow converge subsequentially to a solution of the planar $L_p$-Minkowski problem for $0 < p < 1$. The proof relies on the monotonicity and uniform boundedness of a functional of the flow, called entropy, and on a planar case of the logarithmic Minkowski inequality. (Joint work with A. Stancu.)

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**JIE XIAO**, Memorial University
*Flux-Capacity-Mass*

This talk will address an optimal relationship among entropy flux, electric capacity and graphic mass.

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**DEPING YE**, Memorial University of Newfoundland
*On the monotonicity of affine surface areas under the Steiner symmetrization*

The affine surface area and its extensions are central notions in convex geometry. They have important applications in, for instance, approximation of convex bodies by polytopes, valuation theory, and information theory.

In this talk, I will discuss how the $L_p$ and general affine surface areas change under the Steiner symmetrization. The monotonicity will lead to (stronger) affine isoperimetric inequalities.

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**NING ZHANG**, University of Alberta
*Grunbaum's inequality and centroid position for projections*

Asymptotic geometry expresses various properties of geometric objects as quantities dependent on the dimension. In this talk, a joint work with Matthew Stephen is presented, which gives the best constant for Grunbaum's inequality for projections and interpret Bonnesen and Fenchel's one-dimensional Grunbaum's inequality as an upper bound for the distance ratio.

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**YURIY ZINCHENKO**, University of Calgary
*On the curvature of the central path for linear programming*

Similarly to the diameter of a polytope, one may define its curvature based on the worst-case central path associated with solving an LP posed over the polytope. Furthermore, a continuous analogue of the Hirsch conjecture and a discrete analogue of the "average curvature" result of Dedieu, Malajovich and Shub may be introduced. A continuous analogue of the result of Holt and Klee –a polytope construction that attains a linear order largest total curvature– and a continuous analogue of a d-step equivalence result for the diameter of a polytope may also be proved. We survey the recent progress towards better understanding of the curvature.