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Do Minkowski averages get progressively more convex?

Let us define, for a compact set $A \subset \mathbb{R}^n$, the Minkowski averages of A :

$$A(k) = \left\{ \frac{a_1 + \cdots + a_k}{k} : a_1, \dots, a_k \in A \right\} = \frac{1}{k} \underbrace{(A + \cdots + A)}_{k \text{ times}}.$$

Shapley, Folkmann and Starr (1969) proved that $A(k)$ converges to the convex hull of A in Hausdorff distance as k goes to ∞ . Bobkov, Madiman and Wang (2011) conjectured that when one has convergence in the Shapley-Folkmann-Starr theorem in terms of a volume deficit, then this convergence is actually monotone. More precisely, they conjectured that $|A(k)|$ is non-decreasing, where $|\cdot|$ denotes Lebesgue measure.

In this talk, we show that this conjecture holds true in dimension 1 but fails in dimension $n \geq 12$. We also consider whether one can have monotonicity when measured using alternate measures of non-convexity, including the Hausdorff distance, effective standard deviation, and a non-convexity index of Schneider.

(Joint work with Matthieu Fradelizi, Mokshay Madiman and Artem Zvavitch.)