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Obstructions for primitive solutions to $Ax^3 + By^3 = Cz^2$

It is classical that the only obstructions for homogeneous quadratic equations to have primitive solutions (x, y, z) are local; the Hasse principle holds. The picture for $Ax^p + By^q = Cz^r$ is much more complicated. A result by Beukers shows that if $1/p + 1/q + 1/r > 1$ then the primitive solutions correspond to rational points on finitely many genus 0 curves (subject to certain local conditions). However, as for instance $x^2 + 31y^2 = 5z^3$ shows, obstructions to primitivity are not just local anymore: there is also a class group that can provide obstructions.

For other exponents, obstructions are no longer directly explained by class groups. We will explore some statistics in the case $(p, q, r) = (3, 3, 2)$ This is joint work with Patrick McMahon.