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Integrable structures in 3D hydrodynamic-type systems and differential geometry

We review the theory of 3D hydrodynamic-type systems and Whitham type hierarchies integrable by hydrodynamic reductions method. This approach to integrability is based on the so-called systems of Gibbons-Tsarev type. We explain that this integrable structure can be represented as a certain differential-geometric structure which is defined locally as a family of vector fields $g(p)=\sum_{i=1}^{m} g_i(p,v_1,...v_m)\frac{\partial}{\partial v_i}$ with commutation relations

$$[g(p_1),g(p_2)] = f(p_2,p_1)g'(p_1) - f(p_1,p_2)g'(p_2) + 2f(p_2,p_1)_{p_1}g(p_1) - 2f(p_1,p_2)_{p_2}g(p_2)$$

where

$$f(p_1,p_2) = \frac{1}{p_1-p_2} + O(1)$$

and

$$g(p_2)(f(p_1,p_3)) - g(p_1)(f(p_2,p_3)) = f(p_1,p_2)f(p_2,p_3)_{p_2} - f(p_2,p_1)f(p_1,p_3)_{p_1} + f(p_1,p_3)f(p_2,p_3)_{p_3} - f(p_2,p_3)f(p_1,p_3)_{p_3} + 2f(p_2,p_3)f(p_1$$