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Model theory for topological modules

In 1975, T. A. McKee first introduced a formal language L_t and associated logic suitable for the study of topological structures; this logic was discovered independently about the same time by S. Garavaglia and by M. Ziegler. They established that L_t has a Compactness Theorem, a Lowenheim–Skolem Theorem, and satisfies a Lindstrom Theorem (amongst other things). Later, in my PhD thesis, I showed how to develop Stability Theory in L_t and developed an analogue in L_t for topological modules to the basic "pp-elimination" of quantifiers result for ordinary modules. The logic L_t is thus basically a generalized first-order logic, and while it captures some topological concepts well, it is too weak to capture the full strength of topology, as most of the important concepts of topology are inherently higher order in nature. Thus L_t has not received a great deal of attention since its initial presentation.

My Masters student Clint Enns and I have been investigating to what extent the well-developed model theory of modules might have a useful analogue in L_t for topological modules. In particular, we ask whether there is a natural and useful concept of pure embedding for topological modules, and if so whether there is a natural concept of pure-injective topological module. We cannot expect a competely parallel development, in part because the logic L_t does not "force" mappings between structures to be continuous, and in part because the topological co-product [direct sum] of topological modules does not embed topologically in the topological product.