The chromatic index of a multigraph $M$, denoted by $\chi'(M)$, is the minimum number of colours needed to colour the edges of $M$ such that adjacent edges receive different colours. Shannon (1949), Vizing (1964) and Goldberg (1984) have all established well-known upper bounds for the chromatic index of $M$. In this talk we ask: when is $\chi'(M)$ maximum? That is, when does $\chi'(M)$ achieve a particular upper bound?

Our main result in this talk is to characterize those multigraphs which achieve Goldberg’s upper bound, generalizing a 1968 result of Vizing which characterizes those multigraphs which achieve Shannon’s upper bound. There is no known characterization for those multigraphs which achieve Vizing’s upper bound, however we will discuss some partial results towards this, and address the issue of the complexity of this problem.