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Spanning with Options over a Borel Space

The research question is whether portfolios of finitely many ordinary call options (or put options) allow to hedge any financial claim. The situation considered is a two-date incomplete securities market defined over a metrizable state-space Ω containing uncountably many states of nature. The investigation is limited to securities markets for which the space of contingent claims is identified with an L_p-space, with $1 \leq p < \infty$, on the Borel space $(\Omega, \mathcal{B}, P)$ where $\mathcal{B}$ is the Borel σ-algebra of Ω and P is a nonatomic Borel regular probability.

A claim is an element of the space of contingent claims. The constant function $\mathbf{1}$ represents the riskfree asset payoff. The expression $(s - k\mathbf{1})^+$ describes the payoff of a call option written on an underlying asset s with strike price k. The payoff of a portfolio of the riskfree asset and of finitely many call options is an element of the space O_s defined by

$$O_s = \mathrm{Span}\{\mathbf{1}, (s - k\mathbf{1})^+ : k \in \Re\}.$$

Options are said to span the market if the space O_s is dense in the space of contingent claims. This paper proves that there exist infinitely many underlying assets for which options span a separable L_p-space. In particular, if Ω is also Polish, this collection of underlying assets is dense in the positive cone of $L_p(P)$.