Working Group 2c : Mathematics through the eyes of a child

Ann Anderson, Susan Pitre

Participants:

Ann Anderson, UBC	Susan Pitre, Toronto District School Board
Keith Bauman,	Jane Bennett, Ministry of Ed., ON
Ed Barbeau, U of Toronto	Janis, Kristjanson, MCATA/CBE
Gilles, Laverdure	Susan London McNab, OISE/U of T
Patricia Maxwell, Ministry of Ed., NL	Boon Ong, RMC, Kingston
Florence Roach, Nova Scotia MTA	Sherene Sharpe, South Shore Regional SB
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On this sunny May day, we gather to discuss and share our experiences with, knowledge of and need to know more about mathematics through the eyes of a child. We begin as 13 strangers, although small groups of us or pairs of us know each other from other common experiences and interests: we studied together, we work for the same ministry, we met at another conference.

Who are we then and what has brought us to this CMS Forum and most particularly to Working Group 2c?

As we go around the table and hear names and affiliations, we also hear passion, commitment, and puzzlement with respect to young children's engagement with mathematics. Most of us present work with children indirectly through our work with practicing and/or preservice teachers. In supporting teacher development and learning, we model lessons, provide curriculum, and experience children learning in varied mathematics environments or classrooms. Some of us are here to learn more about the younger child as our experiences and formal education dealt with secondary school mathematical content and learning. Another hopes to have his scepticism, fuelled by teenage apathy toward the subject area, repaired through the energy and engagement of young children in mathematics. Others (most of us) have personal stories to tell of children in classrooms, interviews, one-to-one tutoring, or everyday interactions experiencing/sharing mathematics in creative and unique ways. The collective voice is emerging and the ownership of WG2c shifts to the participants as we, the co-chairs, hoped it would.

As Slide 1 projects onto the screen (See Appendix A), we articulate the goals for WG2c as they were presented in our invitation from CMS and on the CMS Forum website, namely:

1) increase our understanding of characteristics of activities taking mathematics through the eyes of a child in a constructive way, their advantages as well as the obstacles to understanding such activities; and
2) generate specific recommendations to be shared with the wider mathematics teaching and education community.

It was when we invited participants to “unpack” why they came to WG2c that our plans began to morph. The conversation was strong and lively and turn taking soon gave way to dynamic dialogue as individual participants responded to our two guiding questions. What does “mathematics through the eyes of the child” mean to or for you? What are you hoping to accomplish over the next few days, what are your objectives? In Figure 1, we organize the snippets of conversation we captured according to the question we believe it addressed.

Insert Figure 1 here

Sadly, what we failed to note were the rich stories and anecdotes that flowed so easily and captured our imaginations and our undivided attention.

Following coffee break, we decide to move on to the Slide 3 (See Appendix A). If we were to grapple with mathematics through the eyes of a child and delineate activities and professional development to assist others, it seemed appropriate to reflect on our own “abilities” or acumen at seeing mathematics through children’s eyes. To do so, we turned our attention to an excerpt of an interview between a 4 year old and a researcher. Ann proceeds to read aloud and re-enact the interview snippet (Figure 2) that can be found in Towers & Anderson, (199). She brought Jo’s and Amy’s voices and actions into the room, sharing background information, reading the child’s and interviewer’s lines, and synchronizing the writing of the actual exercises with the spoken word. In essence, she shared the sense of the unknown that the interviewer Jo had felt with respect to what to share with a four-year-old, especially with respect to negative numbers. Also, she pointed to the researchers’ lessons learned with respect to what we might see while we are in a situation and what we may see after the fact upon re-viewing the episode alone and with others.

Insert Figure 2 here

The participants picked up on the understanding of the child and conversation revolved around what did she see- was it one and negative 2 or was it ten minus 2 for instance? We discussed how more probing might further help us “see through the eyes of the child”. Just as the interviewer (however not at first) saw that indeed the child’s answer 1 $^{-}2$ was merely one step away from the conventional response of 8, we pondered whether that would be the case for other teachers. If so, what would signal them to probe more?

Other anecdotes of other children’s experiences with negative numbers were shared. We discussed how amazing it seems to have a four-year-old problematize subtraction as Amy did (Figure 2, line 5). Our attention then turned to how easily she picked up a pattern from two examples 7-10=$^{-}3$ and 1-100=$^{-}99$ and applied it to 21-13=1^{-}2 in such a short time frame of this conversation. Of course this example also stimulated rich stories of other examples that participants shared but as they captured our deep attention, we failed to take copious field notes that would permit us to repeat some here.

We offered participants a copy of an article, Bosse (2005), for their overnight consideration, merely indicating that we hoped to use it as a starting point for next day’s opening session.

Although Day 2 for WG2c began with a slightly reduced number in attendance, the energized conversation continued. Ed continued to offer more examples of children's problem solving (as did others) and Janis continued to "knit" seemingly individual comments into the fabric of our deliberations. As we projected Slide 4 on the screen (See Appendix A), we invited participants to share their comments, reflections, reactions to "Akeem's story" Bosse (2005). And, thus our morning began once again with our participants discussing pertinent issues about children's mathematics: what counts as math, what did Akeem really know? The snippets in Figure 3 are taken from notes and were clustered after the fact in an attempt to clarify the substance of the conversations and possibly the tone. Although we make reference to the article for the reader, we provide a brief synopsis so that the comments can be situated in context. In essence, the author, Bosse (2005), taught a summer college algebra course in the evening and one of his students was a single mother who needed to bring her preschool son. Thus, a four-year-old boy named Akeem attended every class. At times when the instructor sensed Akeem was bored, he intervened. For instance, on one occasion he invited Akeem to draw boxes and showed him a method by which a 3-D box emerged when two squares were juxtaposed one square slightly to the left and behind another square and connected the corners. Akeem mimicked what he saw the instructor do, and continued to draw boxes during class as well as at home during the two days between classes. Other iterations occur but most astounding is when Akeem is given full reign of the blank chalkboard in a final class, he proceeds to reproduce graphs that had been the subject of the algebra lessons for the college students. Are you impressed yet? Well it certainly got us talking and a sample of the points we captured are in Figure 3.

Insert Figure 3 here

Thus, the article allowed us to reflect on i) the "gifted" child, ii) the need for multiplicity in our approaches, our tasks, and iii) the need for child-like contexts, for playfulness and creativity. Participants pointed to the importance of honouring the child; they spoke of the privilege to see what they know. In addition, we found ourselves contrasting the role of puzzles, games and play with the starkness of a narrowly conceived "basics" approach.

It was when we returned from coffee, that we turned our attention briefly to Slide 4 (See Appendix A) to consider more directly "how do teachers adjust their points of view to understand their students' points of view?" The theme that emerged seemed to centre on the role of collaboration, of conversation, of talk. Some of us saw the teacher as the one who could add "the math layer", serving as the person who could extract children's mathematics from observations of children and bring it to the foreground. As the few notes captured in Figure 4 indicate, in essence "teacher as listener" seem to underlie our sense of the adjustment that might be needed and yet we felt a need for active communication, such that the teacher also has a role to "talk". In addition, our focus on Akeem pointed towards other pathways to a better understanding of a child's point of view.

Insert Figure 4

We next turned our attention to Slide 5 (See Appendix A) regarding contexts. To get us started Susan shared a description of a Math WebQuest project she had done with fifth and sixth grade French Immersion students. We reconstruct it here so that the reader can be familiar with the catalyst for the discussion that evolved.

As part of my Primary/Junior math additional qualification course, I did a research project based on the use of technology. We know that junior students really love to use their computers; however, its primary use is for chatting and word processing for school. My group decided to see if the use of Internet research for math would stir up more enjoyment of mathematics at such an impressionable age.

This grade 5/6 French Immersion group is very mixed in abilities and cultures. They all have access to a computer and the Internet from home. This is quite an advantage for this research as the students had somewhat developed their computer skills. We decided to see if their interest in math increased with the use of technology – specifically, the use of a mathematical WebQuest.

The main task of this WebQuest was to create an equitable pay scale for NBA players, who are always on the verge of a strike. Each group consists of five team members: an NBA team player, an NBA team owner, a sports agent, a professor of economics and a legal scholar of ethics. Each student had to investigate their role and the perspective that this person would take in salary negotiations. To truly understand the role, the students had to answer questions from the WebQuest with links to answers already in place.

Based on their investigations, each student had to create a spreadsheet that demonstrated their perspective of an equitable salary grid and explain their reasoning. The team then came together to determine which salary grid was the most equitable and to reach a consensus to create a salary grid together. From this class, this was their favourite part!

I conducted conferences with five students from low to high levels of achievement. Throughout the process, these students didn't see that what they were doing was mathematics! It truly was amazing to hear them say this!

Their abilities to create a spreadsheet improved for all of the sample students; however, they still needed to work on their explanation of their work. The improvement was truly impressive.

Snippets of the lively conversation which ensued around what Susan had shared are provided in Figure 5. In addition, participants shared their own rich examples. For instance, Pat shared her daughter's early and continuing interest and passion for real estate and rental income. Her story revealed the meticulous care this child took in searching for property suitable for rental and the investment (time and money) she made when she was quite young. In closing, Pat spoke of her conviction that her daughter could readily interest others through her enthusiasm and knowledge, if a teacher were to tap into that interest or permit her to bring it into mathematics class.

Insert Figure 5

From our sharing and discussion, it became clear that "interests" can be surfaced in multiple ways, including invitations to children to "look" into interests of others including their teacher. It was felt that individual children's interests need to be permitted to grow, to come into play in order to excite others and to motivate themselves. <u>Know</u> the children. Complexity arose as a repeated characteristic and we concluded that when a child pursues real interests, the activities are often adult-like and complex. On the other hand, we are drawn to the child-like and we seem to gloss over the transition question (Slide 5, Appendix A). Maybe there is little press here to concern ourselves with transitions, for when a young child is engaged in an interesting context with mathematics embedded, maybe a transition to the math concepts is not needed; it's all "in the doing".

We reassemble for our last session together. It has been a long exciting day. We turn our attention to support for teachers and discuss what others may need to come to see and appreciate mathematics through the eyes of a child. With Slide 6 beckoning (See Appendix A), we grapple with what recommendations we want to make, "what next?" ideas we have, and what do we want to share with others. As shown in Figure 6, many ideas resurface and get us excited. Our comments were not linear as Figure 6 might suggest but rather we jumped around; ideas about activities spawned possible initiatives which lead to some professional development ideas or when someone made a point about professional development, another raised issues about the activities they share with teachers or others spoke of a way WG2c could support teachers.

Insert Figure 6 here

In particular Ed offers to forward a package of problems to all participants in WG2c, which we in turn agree to share with children in classrooms and report back to each other what we "see" through the eyes of the child. Gilles volunteers to send riddles and puzzles along suggesting we might capture video clips we can share. Others are encouraged to email favourite tasks and children's engagement with them to participants in WG2c. As time and data permit, we talk of placing our portfolio of "mathematics through the eyes of a child" onto a website so they can be more readily shared. For our report to the CMS Forum audience we settle on sharing our immediate initiative (Slide 7) and a broader goal (Slide 8) (See Appendix A).

The ball is rolling! In May after we returned home, Ed mailed each of us an extensive set of problems from CMS notes and Gilles emailed the "Bridge problem" (Figure 7) to all WG2c participants.

Insert Figure 7 here

With the school year nearing its end however, it is not clear that opportunities arose for participants to bring them to classrooms and gather information for the portfolio. Our hope is to surround the problems with commentary about ways in which the problems were shared, what age the children were and in what ways the children engaged with the problem. Thus, the challenge is for all WG2c participants to be vigilant and rise to the challenge to begin our initiative in September so that we may capture "mathematics through the eyes of a child".

References

Bossé, M. (2005) Akeem's Story: Discovering mathematical precociousness, *Teaching Children Mathematics, April*, 430-436.

Towers, J. & Anderson, A. (1998) "The wall that stops the outside coming in": Exploring infinity and other "difficult" concepts with a preschooler, *Early Child Development and Care, 145*, 17-29.

Figure 1: **Initial conversations of WG2c,** Friday, May 6,10:30 – 12:30

Our Wishes as a Working Group : Sound Bytes of Participants

Math through the eyes of a child means:

- How a child learns mathematics – a commonality between all levels
- How do we listen so we hear what they say?
- How do we understand their meaning?
- Expressivity – many ways of saying – oral, symbols, words
  many ways of coping
- If you understand it, you can express it
- Understanding what is being said
- Provide the child with a voice – different levels of articulation
- A connected skill – understanding and explaining
- Mucking around
- Questioning children
- Diversity of students – differentiated instruction
- Expert knowledge – become independent, solve problems themselves
- Junior – too fast into procedural
- Readiness – puzzles, games, are they doing math?

What might we want to accomplish (learn about) from sessions:

- Focused approach – primary, junior, intermediate
- The Kindergarten child
- Early years
  - preK to 1/2
  - K/1 to 2/3

  background, prior experiences, socio-economic, etc.
- How do we help teachers understand math through the eyes of a child?
- Professional Development to support our teachers that's job embedded
- Practical ideas – building capacity, coming back to reflect and pause
- Empowering teachers – confidence
- Working with students at different grade levels

Figure 2: **Overhead of transcript excerpt from interview of a 4 year old.**

Amy: Let's do some math?
Jo: Is this not math?
Amy: Well, I want to do some adding with numbers -that math.

$$\begin{array}{r} 42 \\ \underline{37} \\ 79 \end{array}$$

Jo: Show me a subtraction.

$$\begin{array}{r} 87 \\ \underline{-\ 73} \end{array}$$

Amy: The only problem with take-aways is that I don't know how to do seven take away ten. (Writes seven minus ten in column format) **Can you teach me how to do it?**

$$\begin{array}{r} 7 \\ \underline{-10} \end{array}$$

Jo: So if I was doing seven take away ten I would call it negative three (writing –3 beneath Amy's sum)

Amy: (As she wrote one minus one hundred in column format) **You're going to have to use negative numbers.**

$$\begin{array}{r} 1 \\ \underline{-\ 100} \end{array}$$

Jo: writes –99 beneath the line of calculation
Amy: Now I think I understand more about negative numbers.

Jo: Solve twenty-one minus thirteen, (setting it out in column format as Amy had done)

Amy's response:

$$\begin{array}{r} 21 \\ \underline{-13} \\ 1\ {}^{-}2 \end{array}$$

Towers, J. & Anderson, A. (1998) "The wall that stops the outside coming in": Exploring infinity and other "difficult" concepts with a preschooler, *Early Child Development and Care, 145*, 17-29.

Figure 3: **Responses to “Akeem’s mathematics”,** Saturday, May 7, 9:00 – 10:30

Clusters of participants’ comments:

Always on the look out
Gauss – discovery at age 5
Unsullied insight

Don’t want to miss the next one
Champions – find them when they are young
Stream in the woods – see details underneath

Multiple intelligences
Gender differences
The “what if” scenario – creation of new problems, extensions
Multiple entry points for math – not through the symbolic representation

Qualities of childlikeness – playfulness
Geometrical sense (early on) keep it alive
Time to leave the students “play”

Curriculum pressures
Feel worthy, that they count
The authority lies with the child

Creativity from the child – less creative because of schools
Discover their playfulness, creativity, childlike of math – holistic

Honour and privilege to teach these children
Follow up on how they think: mental computations

Make a point to talk to them
Set him up to do it – mimic

No expectations of Akeem
See and listen to them in their math thinking
Providing a model: a game, discover the math

People get in the way
Don’t try anymore – bored, ignored
Kamii – rediscovering

Bring into alignment child’s mental image and the adults mental image
Look holistically at kids – adults tend to break it down
Not so much about the task, but the interaction between child/teacher and child/math
Deconstruct their learning to move forward : A matter of timing
Worst thing is to say they are right or wrong
Let the creativity lead the way – seek it out
Don’t kill the creative process before it emerges

A lot of things work when put in the form of a puzzle – aesthetics of math
Build whole math curriculum based on games – spiral curriculum and we don’t interfere
Adults that are childlike

Spelling and grammar only and never read literature
Scales and theory and never hear it or play

Literal thinking

How serious do I want to be about math?

Figure 4: **Teachers understanding students' math?** Saturday, May 7, 11:00 – 12:30

- Brent Davis – collective math talk
- Child/child – spark in thought
- Act as a member and an editor of math talk
- Neighbouring interaction – powerful children's mathematical thinking
- Games and puzzles – social – we are social creatures
- Inquiry learning – self contained world
    - Increase the odds with peer learning
- It is not that kids see the math, but the teacher sees the math
- Some do it more naturally
- Transferable skills

Figure 5: **Contexts of interest to children** Saturday, May 7, 11:00 – 12:30

What makes a particular context interesting to children?

- Be like an adult – threshold of the adult world
- Appreciate use of math – Steven Lewis
- Ask the kids – context that is interesting
- MAA – baseball stats
- Doctors without borders – math high school/intermediate
- Familiarity and complexity that goes some place – not predictable
- Depends on area, family life, sensitivity
- Context that interest all – interest others in their interests
- Interested in things that are complex
- <u>Know</u> the children

When and how is the transition made from the context to the mathematics concepts?

- Accept their vision – let them discover
- Applying skills, problems, to existing skills/problems – connecting
- <u>Know</u> the children
- "That's interesting!" over right or wrong
- Novices – look at the surface ; Experts – deep features of the issue
- How do we foster expert thinking? Don't have to be an adult to be an expert thinker

Must some concepts be taught before discussing a particular context?

- back to basics – can they do it?
- Significance of what you're doing
- Does basics mean understanding?
- Cannot agree on what basics are
- Basics are the understanding of why it was invented and how it works
- Novices – look at the surface ; Experts – deep features of the issue
- How do we foster expert thinking? Don't have to be an adult to be an expert thinker
- Be flexible
- Invent their own way
- A learner is a constructor – understanding a concept is putting it in context

- If you have to ask, you'll never know!

Figure 6: **“Teaching Math through the eyes of a Child”** Saturday, May 7, 3:45-5:00

Professional Development

- P.D. – place for teacher – “view students responses and see the math” paradigm
- Teachers can access to children’s responses, as well as analysis of how it fits
- Timing and strategy
- Lesson study-collaborations where we build community amongst children and community amongst teachers
- Teachers communication – chat rooms, email, website
- 2 year for pre-service in Ontario, lesson study – Ministry Teacher’s College
- How we teach pre-service – issues around math anxieties
- How to teach math; how to teach about children’s math
- Life studies – nesting – working together
- Nestedness-teachers work together to study a lesson
- Puzzles, games –mathematically rich-illuminate how they are thinking and how to advance that

Mathematics Activities

- When is it appropriate to go from “raw” to “convention” ?
- Conventions as ‘secondary’?
- Take raw response of the child -transferable?
- “Symbolism”- an orderly presentation; laying out ones reasons
- Kinds of experiences do we provide children-links across?
- When do you do that? Wait until the kids bring it in ?
- Suzuki approach
- Part construction, part evolution
- Self-criticize, self-test-children determine when they’re done
- Math doesn’t occur in isolation
- Math community – communication is essential, fostering all the ways it is communicated, allow the experts to show up
- Personal
- Puzzles, games –mathematically rich-illuminate how they are thinking and how to advance that

Working Group Initiative

- Continuing with mathematically rich games and riddles (video the students)
- Helping students become risk-takers – be creative
- Everyone feels comfortable - the teacher and the student
- What is the outcome? A structured approach to the play
- The follow-up questions asked to see if it is reached
- Problem collection
- Try things in the classroom – begin dialogue about their results/reflections on the occurrence in the classroom
- Nice easy problems that have layers of sophistication
- What age group? Work? Adaptations? Respond? Difficulties? Curriculum? Math payoff?

Figure 7: **Some problems WG2c participants received** after returning home

One of Ed's problems from CMS notes:
Homework!
A Slight Difference
The starred positions are to be filled by the digits from 1 to 8 inclusive to give two four-digit numbers so that the top number exceeds the lower and the difference between the numbers is as small as possible.

* * * *
* * * *

What are the two numbers?

Answer to Homework!
From page A3. If the left digit of the top number exceeds the left digit of the bottom by 2 or more, the difference between the two numbers exceeds 1,000. So we want the top left digit to be one more than the bottom left digit. Now the last three digits of the top number should form a number as small as possible while the last three digits of the bottom number should form a number as large as possible.

5 1 2 3
4 8 7 6

As seen in the rows above, the two numbers are 5, 123 and 4, 876.

NB: Please note that I chose one of the first problems that did not need a diagram to illustrate the types of problems Ed provided.

Gilles' "Bridge" Riddle

The Beatles have a concert in 17 minutes! Unfortunately, they're still in the Underworld recovering John and need to cross over the river Styx using a narrow bridge in order to get to the concert. Since it's dark they need to use a flashlight to cross, but they only have one between the four of them and the bridge can support only 2 at a time. George only takes a minute to get across, Paul takes 2 minutes, Ringo takes five minutes and John takes ten minutes (hasn't done a lot of walking lately). They can only cross as fast as the slowest member. How can they get back to the world of the living on time for the greatest comeback ever?

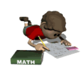
Appendix A

WG2c: “Math through the eyes of a child” Slides

Slide 1

Math through the eyes
of a child

Working Group 2C



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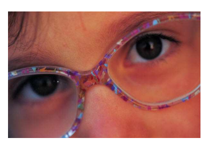
Slide 3

Friday, May 6 10:30-11:30

Can adults see math like a child?



Slide 4

Saturday, May 7 9:00-10:15

How do teachers adjust their point of view so that they better understand their students’ view of the world?

Slide 5

Saturday, May 7 10:45-12:30

- What makes a particular context interesting to children?
- When and how is the transition made from the context to the introduction of mathematical concepts?
- Must some of these concepts be taught before discussing a particular context?

Slide 6

Saturday, May 7 3:45-5:00

Let's generate our responses to…

Teaching Math through the eyes of the child.

Slide 7

Our Initiative

As a group, we will share mathematically rich games, riddles, and puzzles. We will in turn use these with children to collect data on their engagement with the activity.

Our intention is to to build a portfolio of "mathematics through children's eyes".

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Slide 8

Beyond the Initiative

It is hoped that this portfolio will support the working group's efforts in helping all educators to better see the child's views of mathematics.

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