Mathematics Reallocations Document

Introduction

We are living through an era of major conceptual advances for mathematics, and one in which its pervasive influence has radically altered aspects of our society including science and technology. We are in the midst of an information-based technological revolution that is global in scale. Frontiers of knowledge are concentrated in emerging areas that cut across traditional disciplines but have mathematical tools as their common denominator. Mathematical methods are an essential catalyst for progress. Canadian Mathematics has participated fully in this tide of creativity. During the past two decades, both the scientific quality and the relevance of Canadian research in mathematics have increased dramatically. Now Canadian Mathematics has firmly established itself at the international level. It now possesses a critical mass of distinguished senior researchers representing a broad spectrum of key areas in current mathematical research. Canada's role in the mathematical enterprise was examined recently, under the auspices of NSERC, by a distinguished international panel of mathematical scientists. They cited a number of *world class achievements, exceptionally high level groups and topics which in Canada have recently had special developments*. They concluded that *Canadian mathematicians are participating at a high level in the thriving enterprise of mathematical research.* As another index of impact, the International Mathematical Union (IMU) recently invited Canada to apply to join Group V, its highest ranking.

Canada has produced and recruited a large number of very talented junior mathematicians. New developments are well represented in the main areas of Canadian mathematics: number theory (representation theory, diophantine equations, arithmetic geometry); geometry (symplectic geometry, gauge theory); analysis (C*-algebras, nonlinear PDEs); mathematical physics (superconductivity, quantum theory); and probability (superprocesses, statistical mechanics). This decade has also witnessed the emergence of new areas of science and industry based on innovative applications of mathematical theory: biological modeling (PDEs, probability); computational software (fluid mechanics); cryptography (elliptic curves); financial mathematics (stochastic control, PDEs); materials science (PDEs, numerical analysis); and telecommunications (wavelets, probability).

New directions and initiatives, with major significance for the future of the discipline, have originated in this decade. The growing emphasis on outreach is one striking new development in Canadian Mathematics during the past five years. During this time, the mentality of outreach has been firmly established in the mathematical community and industrial collaboration has expanded at a rapid pace. Mathematics is leveraged as never before. The Review of Canadian Mathematics commented that *Canadian mathematicians have focused considerable attention in recent years on the advantage of, and techniques for, outreach to other disciplines and to the community at large.*

Another significant development of the past five years has been the emergence of the research institutes for the mathematical sciences as a national presence in Canadian mathematical life. The mathematical institutes (the CRM, the Fields Institute, PIms) represent an important addition to Canadian mathematics. They have a substantial record of achievement in organizing and sponsoring an amazing diversity of innovative scientific research. In this respect the Review of Canadian Mathematics commented that *the Fields Institute is a major success story for science in Canada*. As well, the institutes are the prime agents of mathematical outreach with a growing number of very successful industrial collaborations. The NSERC funded *Network for Computing and Mathematical Modeling,* developed under the leadership of the CRM, exemplifies such effective collaboration. In addition, the institutes are demonstrating a strong sense of national purpose and have achieved a new high level of cooperation in their promotion of the mathematical sciences. The establishment of PIms was the crucial step in developing a national research network capable of supporting and integrating the research, outreach, and training activities across Canada.

Summary

- We will demonstrate the impact and scientific quality of Canadian mathematics. Canada has a number of internationally renowned mathematicians. A large group of Canadian mathematicians are involved in research areas of current and future impact at a world class level. In particular, a number of significant mathematical results have been produced by Canadian mathematicians.
- We will demonstrate the increased relevance of Canadian mathematics via its growing interactions with science and industry. This follows from the increased demand for mathematically trained professionals, the significant increase in outreach, especially through the institutes, dramatic increase in leveraging of individual grants, and the quantity and quality of interdisciplinary work.
- We will establish a rationale for increased GSC funding. This is based on (1) the number of strong new young researchers, especially in emerging new fields and those with a more applied emphasis; (2) the significant increase in graduate students and demand for graduates; and (3) the strong cadre of senior people combined with few top grantees leaving the system in the next decade.
- We will demonstrate the growing impact and stature of the mathematical institutes. Their record of supporting quality scientific activity from a broad range of core and interdisciplinary subjects in the mathematical sciences is exemplary. They have had remarkable success in establishing contacts with the industrial/financial world. They will be playing an increasingly vital role in training of HQP. This is the basis for a strong case for the *Type II proposal* to provide more funding for the three major research institutes. This is critical to allowing mathematics to fully develop its scientific potential as well as reach outside to industry and other disciplines.

For clarity, this document is divided into two distinct parts, corresponding to the two distinct funding proposals that it contains. Part A focuses on the activities and accomplishments of individual mathematicians and their need for more GSC funding, while Part B fulfills the same role for the Institutes.

Part A: Mathematics GSCs 336 and 337

Section A.I: Scientific Activity

This section provides a selective overview of recent Canadian mathematical research. There is a focus on the recent work of some of the top young researchers and some of of the most significant senior researchers. We begin with a discussion of several key characteristics of mathematical research which provide a fuller understanding of the nature of the achievements described below.

(i) Mathematics is the conceptual framework for much of science and technology; but, first and foremost, it is an autonomous discipline with its own internal life. The internal development of mathematics is dominated by a drive towards structure, abstraction, and the creation of powerful conceptual tools. This pursuit is often long term and incremental. Conjectures based on relevant examples play a very significant role: structures are anticipated long before they are established. Projects in mathematics span a spectrum from problems which have been around for centuries (e.g., Fermat's last theorem) to those responding to problems of the day (e.g., pollution control).

(ii) One thrust of mathematics concerns interconnections between seemingly disparate scientific and technological areas. Such developments often have a major impact on other disciplines. Mathematics focuses on methods; a chemist studying chemical reactions uses similar mathematical tools to an ornithologist investigating the migration of birds, or to an economist describing fluctuations of financial markets. Ideas developed in number theory have led to elliptic curve cryptography, at the core of emerging telecommunications technology. Harmonic analysis and wavelet analysis form the basis of emerging video data compression standards.

(iii) The interplay between theory and applications is one of the dynamics of mathematics. The most applied practitioners are found everywhere, not only in mathematics but also in other sciences, engineering, and industry. They work on problems arising directly in these disciplines or in industry, and they apply to them the full range of conceptual and computational tools developed by mathematicians. Conversely, these applied problems are often catalysts for new theoretical developments.

Awards. The growing number of awards and distinctions awarded to GSC 336/337 grantees provides strong evidence of the impact of Canadian mathematical research. One of the very highest honours to be accorded a mathematician is to be invited to speak at the International Congresses of Mathematicians (ICM) which are held every four years. The already mentioned initiative by the IMU to elevate Canada to its highest ranking reflects the fact that ten mathematics grantees have been invited to deliver major ICM talks at the last three meetings while three more have been invited for 1998. Only six countries had more speakers. Additionally, in the past decade, four mathematicians have been awarded Steacie Fellowships, seven have been awarded Sloan Fellowships and ten have held Killam Fellowships. Also in the past ten years, 19 mathematicians from GSC 336/337 have been elected Fellows of the Royal Society of Canada; and one has been elected to the Royal Society. In the past three years, eight Canadian mathematicians have delivered plenary talks at meetings of the American Mathematical Society (AMS).

We will organize our discussion by subdisciplines. It is important to recognize, however, that there are no real boundaries and there is much overlap between the areas. Each of the main areas to be surveyed encompasses a spectrum of activity ranging from the theoretical to the applied.

Number Theory is one of the leading areas of Canadian mathematics. Number theory has intimate connections with algebra, analysis, and geometry, and much of its modern development has arisen out of exploring these connections. One of the most important developments in modern number theory concerns its growing ties to representation theory and to the theory of automorphic forms. The Canadian mathematician Robert Langlands (IAS Princeton) outlined a visionary program for establishing a comprehensive framework of 'automorphic representations'. J. ARTHUR (Toronto, FRS, FRSC, ICM'82, ICM'98, Tory Medal) is the world leader in the famous Langlands program. His work is a combination of harmonic analysis, automorphic representations, Lie group theory, geometry and number theory. He is best known for his profound generalization of the Selberg trace formula and his work with Clozel on base change, central results in the ongoing development of the Langlands program. His current work centres on classifying the automorphic representations of classical matrix groups.

The classical area of analytic number theory has become an area of major focus during the past decade. Canada has several outstanding representatives. J. FRIEDLANDER (Toronto, FRSC, ICM'94), together with Andrew Granville, disproved the conjecture of Elliott and Halberstam on the equidistribution of prime numbers. Together with Iwaniec he made a spectacular advance by proving that there are infinitely many primes which are the sum of a square and a fourth power. In a series of striking papers, C. STEWART (Waterloo, FRSC, Killam) and collaborators have obtained best possible upper bounds for the number of solutions of polynomial equations. In particular, he showed that S-unit equations usually have few solutions, but under certain circumstances can have surprisingly many. D. ROY (Ottawa) is a rising star whose recent result with Thunder on Siegel's lemma represents a breakthrough in transcendence theory.

Another area of major activity for the past decade has been the study of L functions and elliptic curves, subjects dealing with deep interconnections between number theory and algebraic geometry. K. MURTY (Toronto, FRSC, Steacie) has done recent important work concerning the Tate and Hodge conjectures for L functions attached to arithmetical varieties. He and R. MURTY (Queens, FRSC, Steacie) have studied the non-vanishing of L functions, a step towards the proof of the Birch-Swinnerton-Dyer Conjecture for the theory of elliptic curves. R. Murty has, in highly original fashion, brought analytic techniques to bear on a number of important questions coming from algebraic number theory, especially from the theory of elliptic curves. H. DARMON (McGill, Sloan), an outstanding young researcher, has obtained striking results on Diophantine equations. D. BOYD (UBC, FRSC, Steacie prize) has studied Mahler measures and their connections with ergodic theory, K-theory and L functions.

Algebra. Canadian algebraists include a very distinguished group of researchers with significant links to a number of other areas. R. MOODY (Alberta, FRSC, Steacie, Wigner medal) is famous for 'Kac-Moody algebras'---these play a pivotal role in the theory of Lie algebras and have important applications in theoretical physics. Several algebraists have strong connections to the representational aspects of number theory: V. SNAITH (McMaster, FRSC) is well known for his papers and books on explicit Brauer induction as well as foundational work on K-theory in algebraic topology; while A. WEISS (Alberta) has recently solved two noted conjectures, the Zassenhaus Conjecture for group rings and the Strong Stark Conjecture of Chinburg for abelian extensions in number theory. The research of V. PLATONOV (Waterloo, ICM'74, ICM'78, Humboldt) is concerned with a wide range of problems in the theory of groups and algebras, especially algebraic groups. He recently solved certain cases of the rationality problem for group varieties. A. JOYAL (UQAM, FRSC, Killam) is noted for his application of category theory concepts to a wide range of interests. The work of M. MAKKAI (McGill) is an innovative blend of model theory and category theory. S. TODORCEVIC (Toronto, ICM'98) has revolutionized Ramsey theory for infinite sets, and has found remarkable applications to topology, ergodic theory, Banach spaces and algebra.

Geometry and Topology pervade almost all branches of mathematics, and they also have diverse and far reaching interactions with biology, chemistry, computer graphics, and physics. The surprisingly strong interaction of theoretical physics with geometry and topology is the most exciting recent development. Gauge theory has revolutionized low dimensional topology, while there has been explosive growth in symplectic geometry inspired by problems originating in physics.

Symplectic and contact geometry are among the most fascinating new research fields in the physical sciences. The work of F. LALONDE (UQAM, FRSC, AMS) has been crucial in the understanding of symplectic capacities and provided some of the first results applicable to all varieties. With his main collaborator, D. McDuff (FRS), who will give a plenary talk at ICM'98, he classified the first family of symplectic manifolds in dimension four. L. JEFFREY (McGill, Sloan, AMS), a rising star, has given with her collaborators F. Kirwan (FRS) and A. Weitsman proofs of the famous Witten formulas. She has made fundamental contributions to the relation between quantization and symplectic reduction in the abelian case. Other outstanding young symplectic geometers are E. MEINRENKEN (Toronto), who established the factorization entering in the Verlinde formula and proved the commutation of quantization and reduction in the positive-definite case, K. BEHREND (UBC) with important contributions to the foundations and computations of quantum cohomology, and B. KHESIN (Toronto, Sloan, Aisenstadt) for his work on topological hydrodynamics and infinite dimensional integrable systems.

In gauge theory, at the interface between field theory and geometry, J. HURTUBISE (McGill, AMS Centennial Fellow, AMS) and his collaborators gave a proof of a long standing gauge theoretic conjecture of M. Atiyah and J. Jones. His research gave fundamental insights on various moduli spaces of maps. This exciting field is represented as well by I. HAMBLETON (McMaster) who also works on group actions using a large range of mathematical techniques.

E. BIERSTONE (Toronto, FRSC, AMS) and P. MILMAN (Toronto, FRSC) are world leaders in the classical field of singularity theory. They have given a constructive proof for one of the most famous theorems in modern mathematics, Hironaka's desingularization in characteristic zero for which he was awarded a Fields Medal. This group also includes A. KHOVANSKII (Toronto, ICM82) who created the field of *fewnomials,* a theory playing a foundational role in complexity theory. Relating combinatorics to algebraic geometry, he showed how discrete invariants of an algebraic variety can be computed by simple summation formulas over the integral points of Newton polyhedra.

As part of Fields Medalist W. Thurston's program to classify 3-dimensional manifolds, S. BOYER (UQAM) and X. ZHANG (a UBC graduate) have recently completed a series of papers on exceptional surgeries providing the best upper bounds for specific non-hyperbolic surgeries. K. MURASUGI (Toronto, FRSC) and D. ROLFSEN (UBC) are other well-known knot theorists. The former was recently awarded the Japanese Mathematical Society Fall Prize for his proof of the 100 year old Tait conjectures in knot theory.

There is a very active team in differential geometry and topology at McMaster including M. MIN-OO,

A. NICAS and M. WANG. The dominant theme in this group is Riemannian geometry and its connection to physics. New striking results on functionals on Riemannian metrics on manifolds, an area related to quantum gravity, have been obtained by A. NABUTOVSKY (Toronto). J.Y. CHEN (UBC) has generalized Gromov's theory of pseudo-holomorphic curves to other spaces of maps.

There is a strong group in Ontario working in the core subject of homotopy theory. S. HALPERIN (Toronto, FRSC) is an outstanding topologist and the leader of an international program to understand the rational homotopy of finite complexes. R. JARDINE (UWO, Sloan) has made very significant contributions to algebraic K-theory and simplicial homotopy theory.

Functional Analysis has a strong presence in Canada, particularly in the area of operator algebras. Operator algebras can be used to encode diverse phenomena such as dynamical systems, groups representations and quantum mechanics. The subject represents a blending of analytic, algebraic and topological techniques. G. ELLIOTT (Toronto, FRSC, ICM'94, Killam) is the author of one of the most exciting developments in the subject in decades. He has made a radical proposal that a huge class of C*-algebras can be classified by simple, reasonably computable invariants. Moreover he proved the prototype theorem establishing his conjecture in a special case. Dozens of researchers have joined his quest. D. HANDELMAN (Ottawa, FRSC, Steacie, Killam) with Effros and Shen classified K_0-groups of an important class of C*-algebras. His work with Boyle on the spectrum of nonnegative matrices, which they developed to study dynamical systems, won them the Hans Schneider Prize. K. DAVIDSON (Waterloo, FRSC, Steacie, Killam) is well known for his work on non-self-adjoint operator algebras and single operator theory. He showed that two nests (chains of subspaces of a Hilbert space) are similar if and only if there is merely a dimension and order preserving map between them. This leads to basic structural results for triangular algebras. M.D. CHOI (Toronto, FRSC) has made deep connections between (completely) positive maps and the structure of C*-algebras.

There are a number of strong young researchers in the area. An outstanding one is I. PUTNAM (Victoria, Aisenstadt) who, with T. GIORDANO (Ottawa) and C. Skau, recently used C*-algebras and topological invariants to establish the beautiful and remarkable result that two minimal Cantor dynamical systems have the same orbits if and only if they have the same invariant measures. M. KHALKHALI (Western) is another promising young operator algebraist who works on cyclic cohomology. A. NICA (Waterloo) is a very strong new appointment working on non-commutative probability theory in operator algebras.

Geometric functional analysis focuses on Banach spaces. This area has been stimulated by a number of problems that remained unsolved for almost 50 years. The contribution of N. TOMCZAK-JAEGERMANN (Alberta, FRSC, ICM'98, Killam, AMS) was crucial to the positive solution of a famous problem raised by Banach in the 1940s: Is Hilbert space the only Banach space isomorphic to all of its infinite dimensional subspaces? A. LAU (Alberta) works on noncommutative harmonic analysis, and has obtained interesting characterizations of amenability of locally compact groups.

Partial Differential Equations describe states of systems, physical, biological, economical, varying in space and time. Canada has one of the strongest groups in linear PDEs worldwide. V. IVRII (Toronto, ICM'78, ICM'86) established the 1909 conjecture by H. Weyl concerning the distribution of eigenvalues of Laplace operators. Ivrii and I.M. SIGAL proved the longstanding Scott conjecture on the structure of ground states of large molecules. M. ZWORSKI (Toronto, Sloan) has many new results on geometric scattering theory, singularities of non-linear waves diffracted by a convex body, and distribution of resonances.

A striking new theory of integrable non-linear equations, with connections to symplectic geometry, has been developed by O. BOGOYAVLENSKIJ (Queens) and N. KAMRAN (McGill, Aisenstadt). Kamran has provided a classification of hyperbolic equations in the plane. J. TOTH (McGill) has proved important results in the semi-classical behaviour of eigenfunctions of quantum integrable systems.

P. GUAN (McMaster, Sloan) works on PDEs of degenerative type. He and Sawyer completely solved the regularity theory for the oblique derivative problem. N. GHOUSSOUB (UBC, FRSC, AMS) has developed new variational methods to deal with borderline PDEs like those involving the critical Sobolev exponent or with perturbations from symmetry. He and talented young analyst C. GUI (UBC) have given an affirmative answer to De Giorgi's conjecture in low dimensions regarding the uniqueness of transition profile.

Mathematical Physics. Canada has an internationally eminent group in mathematical physics, which makes fundamental links to many other areas: algebra, geometry, probability, and in particular PDEs. I.M. SIGAL (Toronto, FRSC, ICM'90, Killam, AMS, Synge Prize) and Soffer proved the 30 year old conjecture of asymptotic completeness in many particle scattering theory. This conjecture states that, left to its own devices, a system of particles after a period of time will break up into stable, independently moving fragments. Bach, Froehlich, and Sigal developed a mathematical theory sitting at the very origin of quantum theory---the emission and absorption of radiation by systems of particles, such as atoms and molecules. J. FELDMAN (UBC, FRSC, ICM'90, Synge Prize) derived a rigorous variant of a celebrated renormalization group method that earned K. Wilson a Nobel Prize. He, Trubowitz, and others have nearly completed one of the most ambitious programs in modern mathematical physics, laying out a mathematical theory of superconductivity. The theory of Fermi surfaces in many particle systems is a notable spin-off of this program.

Many young Canadian mathematical physicists are making their mark on the international scene. C. ALBANESE (Toronto) has deep results on random differential operators and statistical mechanics of phase transitions, while the work of L. SECO (Toronto, Sloan) with Fields medalist C. Fefferman on the theory of large atoms was discussed in the Encyclopædia Britannica. Their proof of the Dirac-Schwinger conjecture for ground state energy combines techniques from special asymptotics, PDEs and ODEs, analytic number theory and computer assisted analysis. R. FROESE (UBC) is co-author of one of the central theories related to the Schroedinger equation, the Froese-Herbst theory of spectral instability.

Applied Math. Most problems in the physical sciences and many in the biological sciences can be formulated in terms of nonlinear partial and ordinary differential equations. The mathematical analysis of these equations and their solutions (e.g., regularity, bifurcation and asymptotic behaviour, numerical approximations, optimization) can often provide insight not only on mathematical issues but also on the physical phenomena under consideration, thereby having a substantial impact on industrial applications. There has been an growing trend in Canada over the past ten years, through the hiring of young talented researchers, towards this type of interdisciplinary mathematical research.

In the field of continuum mechanics, M. FORTIN (Laval) is a leading figure on the numerical analysis of fluid and solid mechanics problems using finite element schemes. He has made key contributions to convergence analysis and optimal error estimates. The software applications of these advances are described in the Interactions section. J. HEYWOOD (UBC) provided, with Rannacher, the first full convergence analysis of finite element schemes for the Navier-Stokes equations. These theoretical advances are the basis of an acclaimed CFD solver. Key results of R. ILLNER (Victoria) include a global existence and uniqueness result for the Boltzmann equation in the limit of large mean free paths and the first complete convergence analysis for particle methods for the Boltzmann equation. T.B. MOODIE (Alberta) was the first to introduce strain energy concepts into cardiovascular research. He solved a fifty year old problem by providing a frame invariant form of the constitutive equations of thermo-elasticity. An outstanding contribution to fluid mechanics was the rigorous derivation by the young researcher J. QUASTEL (Toronto), together with Yau, of the Navier--Stokes equation as the continuum limit of a many particle stochastic system.

There are many emerging areas involving applications of mathematics and many talented younger researchers working in these areas. The study of self-focusing phenomena for the nonlinear Schrodinger equation of nonlinear optics came to a decisive conclusion with the work of C. SULEM (Toronto), Papanicolaou and their collaborators. M. WARD (UBC, Aisenstadt) has made important contributions to metastable phenomena in reaction-diffusion equations with materials science applications. For this work, he gave a plenary talk at the International Congress of Applied and Industrial Mathematics (ICIAM) in 1995. L. BRONSARD (McMaster) has made key contributions to the rigorous study of interfacial dynamics for the Cahn-Hilliard equation and related problems in materials science. Dendritic and crystal growth has been studied by J.J. XU (McGill), and he has received funding from the U.S. space agency NASA for his work. A. PEIRCE (UBC) has made key contributions to the computational analysis of problems in rock and fracture mechanics and has research ties to Schlumberger. Finally, C. GUI (UBC) has recently given a complete characterization of multiple peak solutions for nonlinear singularly perturbed Neumann problems that arise in the study of pattern formation in biology.

Mathematical biology has grown enormously over the past 10 years in Canada. R. MIURA (UBC, FRSC, Guggenheim), co-founder of the theory of solitons, is a leading figure in mathematical properties of excitable media and has analyzed bursting electrical behavior in pancreatic cells and electrical properties of cortical neurons. W. LANGFORD (Guelph) is a leading expert in bifurcation theory. He has studied such diverse topics as the transition to turbulence in the famous Taylor--Couette experiment in fluid mechanics, resonance phenomena for flow-induced vibrations, chaotic mode interactions, and instabilities in biological systems. J. WU (York) has made extensive studies of bifurcation behavior for delay-differential equation models in mathematical biology.

In optimization, M. DELFOUR (Montreal, FRSC, Killam) is a leading expert in the modeling, optimal design and control of engineering and technological problems. Recently, he has developed intricate theoretical and numerical tools for problems in shape optimization. U. HAUSSMANN (UBC) is a leading figure in optimal control for stochastic differential equations with applications to math finance. A. LEWIS (Waterloo, Aisenstadt) has provided, with J. BORWEIN, key convergence results for maximum entropy methods.

Computer Mathematics. There are a number of significant interactions with computer technology in the rapidly developing field of computational number theory. H. WILLIAMS (Manitoba, Killam) has made a number of advances in primality testing and public key cryptography. J. BORWEIN (SFU, FRSC, Chauvenet prize, APICS/Fraser prize) and P. BORWEIN (SFU, Chauvenet Prize) have made striking advances in algorithms for computational number theory, notably with their amazing and efficient way of computing specific digits of π. In his work in convex optimization and nonlinear analysis J. Borwein also has developed projection algorithms for solving convex feasibility problems.

Discrete mathematics is linked inherently to computer science. The mathematics of computation and discrete modeling combined with the tremendous increase in computational power has made previously intractable problems of science and industry accessible. An essential ingredient is the use of powerful new mathematical algorithms. The current evolution of discrete mathematics is towards the use of increasingly sophisticated mathematical structures. The application of the theory of elliptic curves to cryptography by R. MULLIN and S. VANSTONE (Waterloo) is an outstanding example of the power of such sophisticated mathematical structures. Certicom, in Mississauga, makes digital cryptographic OEM devices, based on their work. Vanstone is one of *two* new NSERC industrial chairs in cryptography which have just been established as part of a Centre for Applied Cryptographic Research at Waterloo.

The discrete mathematics community has a pattern of wide distribution, reflecting its orientation towards applications in computer science and engineering. Among the discrete mathematics funded by GSC 336/337, there is special strength in algebraic combinatorics, centred at Waterloo and UQAM. D. JACKSON (Waterloo) is investigating embeddings of graphs in surfaces, a basic structure used to understand a wide variety of phenomena in algebra, analysis, and topology. C. GODSIL (Waterloo) deals with association schemes and their relations to graphs, codes, and designs. C. REUTENAUER (UQAM) is working on noncommutative rationality and recently solved, with G. Duchamp, a conjecture of A. Connes on operator algebras of the free group. In graph theory, B. ALSPACH (SFU) is involved in a major development in the relatively new area of matching designs, as well as analysis of Cayley graphs. A disproportionate number of international technical journals in discrete mathematics are edited in Canada.

Probability. Over the past 20 years, new ideas in probability have been driven by interdisciplinary and industrial applications. Connections with mathematical biology, finance, statistical mechanics, computer science, and telecommunications have led the subject in many new directions. Canada has an extremely strong group of researchers in probability and is at the forefront of many of these developments.

The interconnected areas of stochastic PDEs, interacting particle systems, and superprocesses or branching measure-valued diffusions have been some of the most active and exciting areas of probability in recent years and are ones in which Canadian mathematicians have been at the forefront. The recent emergence of these probabilistic methods in mathematical biology and, in particular, in population genetics and mathematical ecology is one of the important developments in the subject. The foundational work of J. WALSH (UBC, FRSC) in stochastic PDEs and branching particle systems was pivotal to the development of these areas. E. PERKINS (UBC, FRSC, ICM'94, Steacie) is a world leader in the field of superprocesses and has helped pioneer many of the fundamental concepts. A very significant advance was his development of a theory of stochastic calculus allowing one to set up strong equations for superprocesses driven by super-Brownian motion.

In statistical mechanics, one of the most important recent results was the application by G. SLADE (McMaster, ICM'94) of the lace expansion (mathematical physics) to combinatorial and probabilistic problems, such as polymer models (self-avoiding walks, lattice trees) in chemistry and percolation. He has more recently established important fundamental links between these polymer models and superprocesses.

Disordered media arise in physics in numerous situations and has led physicists to study random walks on fractal-like sets. This has motivated the study of a variety of new stochastic processes which require new techniques. M. BARLOW (UBC, ICM'90) is the leading expert in the world in this area and has had a major impact on the Japanese probability community in particular.

Section A.II: Interactions

The Review of Canadian Mathematics notes the inextricable links of mathematics with science and technology and asserts that it is these links *which are likely to shape the future organization of the mathematical enterprise*. In particular, as these links grow stronger, they will increasingly affect the nature of mathematical research and the nature of graduate education in mathematics.

The growing scope of such interactions is the most striking new development in Canadian Mathematics during this decade. There is a significant and increasing emphasis on mathematical research oriented towards interdisciplinary activity. Moreover, this decade has witnessed a tremendous growth of outreach. The change in mindset as a result of these activities is the most fundamental new factor in this regard. The growth of contacts between mathematics and the worlds of finance and industry is a totally new feature of Canadian mathematics in the 1990s. Five years ago, there were very few contacts. Now contacts directed towards the private sector are extensive. This is an area of great future potential for both research and graduate training.

One also should record that increasingly well-defined targets for such outreach have emerged during this period. Mathematical outreach activities have primarily been directed towards a number of *innovation industries*: finance, information processing, transportation and telecommunication, and health and environmental science. These are all areas in which significant challenges have arisen involving questions in modelling, simulation, data analysis and pattern recognition, and control and analysis of large scale systems. Solutions to these problems involve the use of innovative and sophisticated mathematical tools. Input from mathematicians is increasingly being sought in these sectors.

Leveraging. One compelling index of the growth in mathematical interactions is leveraging data for top grantees from GSC 336/337. Table 11 of DD shows a dramatic increase in the leverage ratio---from a factor of 0.27 (1984) to 1.19 (1995). This increase far exceeds that of most other GSCs. This data does *not* include leveraged funds obtained by the mathematical institutes---inclusion of this would double the leveraging ratio.

Another significant index of the rise in mathematical interaction is the number of smaller, typically university-based, institutes which have been founded (almost all in this decade) with a mandate to target applications and to build bridges between the mathematical sciences and areas of science and industry. Among such institutes are: the Institute of Applied Mathematics (IAM) at UBC, the Center for Experimental and Constructive Mathematics (CECM) at Simon Fraser, the Applied Mathematics Institute (AMI) at Alberta, the Institute of Industrial Mathematical Sciences (IIMS) at Manitoba, Le Laboratoire de Combinatoire et d'Information Mathematique (LACIM) at UQAM, Le Centre Interuniversitaire de Calcul Mathematique et Algebrique (CICMA) at Concordia, Laval, and McGill, and Groupe Interdisciplinaire de Recherche en Elements Finis (GIREF) at Laval. Several such activities will be described below.

There is a long and fruitful tradition of individual mathematicians interacting with people in other disciplines. This has intensified in this decade. The previous survey of scientific activity identified many mathematical fields and researchers with substantial ties to other areas of science. We now turn to an explicit description of the outreach activity of these mathematicians. As a consequence of increased interdisciplinary activity in the past decade, there has been a dramatic increase in attention to mathematical applications in the innovation industries. There have been exciting developments in the emerging areas of algorithms and computing, cryptography, financial mathematics, and telecommunications.

- In the area of algorithms and computing, M. FORTIN and his consulting group, GIREF, have produced sophisticated computational software packages to study various industrial problems in fluid and solid mechanics. GIREF at Laval has the mandate of promoting and developing mathematical techniques for the engineering sciences. Joint development of software by the two disciplines is a major activity of GIREF. This group has many industrial contracts, including the Bombardier corporation. It annually organizes the successful *Journee des elements finis.*

- Cryptography is a rapidly developing area of the past decade. An outstanding success story in this area are the industrial applications of the Waterloo mathematicians R. MULLIN and S. VANSTONE. In 1985, they and others formed a company to market a computer chip implementing their cryptosystem based on elliptic curves. Certicom now employs roughly 75 people, including 18 mathematical/engineering PhDs. Among its customers are Revenue Canada, IBM, Motorola, and various banks.

- Financial mathematics has become a major area of focus in Canada during this decade. Graduate programs have been established at many universities, and the Fields Institute has made financial mathematics a priority of its outreach program. The Clark-Haussmann formula (U. HAUSSMANN, UBC) in stochastic calculus, is one of the fundamental tools used in the pricing of derivatives. Theoretical advances in stochastic analysis over the past 25 years have revolutionized options trading (e.g., the celebrated Black-Scholes pricing formula for which the Nobel Prize in Economics was recently awarded to Canadian M. Scholes). The ideas used to price options during market uncertainty now are being adapted to the insurance and reinsurance industries. C. ALBANESE and L. SECO at Toronto have established contracts with Algorithmics Inc., and are applying the theory of PDEs to mathematical finance.

- Telecommunications is an area in which Canadian expertise is poised to make significant contributions. Queueing theory offers such possibilities. Canada is at the forefront of ATM switching technology. Fundamental methodological progress could make an important contribution to one of the country's largest industries. A concerted joint effort by the three mathematical institutes is being planned. D. MACDONALD (Ottawa) has initiated a number of telecommunications projects with Nortel, Newbridge, and BNR based on his research in switching analysis and networks.

- The areas of harmonic analysis and approximation theory from classical analysis have had a major visible impact through the development of wavelet and fractal analysis for use in telecommunications, image compression and transmission, and computer aided geometric design, plus rapidly developing applications to many other areas. Fundamental criteria for the convergence and stability properties of the wavelets, multi-wavelets and subdivision schemes used in these applications were given by R.Q. JIA (Alberta) and his collaborators. E. VRSCAY (Waterloo) is director of the Waterloo Fractal Compression Project, devoted to the theoretical and practical developments of fractal image compression.

- J. BORWEIN as Director of the CECM at SFU has organized many projects straddling telecommunications and computing theory. In collaboration with the BC Cancer Agency, he has developed projection-type algorithms for signal reconstruction. The CECM explores and promotes the interplay between conventional mathematics and modern computation. It is supporting research projects in symbolic computation, numerical computation, complexity, collaborative network technology, digital information, and visualization. From its base funding, CECM leveraged $550 K in outside funds in 1996--97.

- The IIMS at Manitoba has focused on building links between university research and industry since its establishment in 1991. It is coordinating a new interdisciplinary Master's Degree in Mathematical and Computational Sciences tailored to industry employees. It has organized recent conferences on applications of linear algebra, on porous materials and on fluid dynamics.
- M. DELFOUR consults with industry on a variety of optimization problems. He has a long established project with Industry Canada concerning the distribution of radio frequencies in major Canadian cities. J.C. CLEMENTS (Dalhousie) has organized, or participated, in projects with CBCL (designing ship hulls), the Medical Research Council (cardiac potential mapping), and the DND (optimizing aircraft movements). Through his study and simulations with non-local PDE models of microelectronic devices, W. ALLEGRETTO (Alberta) plays a major role in the development of a cantilever-in-cantilever micromachined sensor device capable of detecting mass changes of .1 nanogram in research sponsored by the Canadian Microelectronics Corporation.

Section A.III: Highly Qualified Personnel

NSERC data gives strong evidence of two patterns in mathematical graduate education: dramatic growth, and excellent employment prospects for the students trained in these programs. The discussion of these patterns will take place in the section on funding needs. In this section we focus on initiatives which have arisen in the mathematics community regarding graduate training.

There are strong indications that interdisciplinary graduate training will be the area where much of the future growth is located. The number of emerging areas of research in this domain and the very positive job market for mathematical science graduates in high tech industries both suggest such growth. As an example of a very active ongoing program in interdisciplinary training we cite:

- For the past 25 years, the Institute of Applied Mathematics has provided the University of British Columbia with a very effective multidisciplinary, inter-departmental applied mathematics presence. It currently has 46 faculty members from 13 different departments. These faculty members supervise 35 graduate students, most of whom are registered in the Department of Mathematics.

Interdisciplinary training in mathematics is undergoing rapid evolution. Many such programs have emerged in the past few years. Very successful Masters programs in Financial Mathematics have been initiated at Alberta and Waterloo, and others are being actively planned (Calgary, UBC, Toronto). Masters programs in Industrial Mathematics also have begun to receive serious attention, and it is evident that a number of such programs will emerge in the next few years. Manitoba has just instituted a new interdisciplinary Masters Program tailored to upgrade the mathematical skills of those already working in industry. Memorial has established a Masters Program in Computational Science.

Interdisciplinary research and training has extra needs and, as a result, is more expensive than other areas of mathematical training. Large scale computing facilities are one obvious requirement. As well, there is a need for providing students with continuous exposure to mathematical challenges and to innovative research arising from other disciplines and the industrial sector. Research in the industrial setting, in particular, requires face to face contact with representatives from industry. Funding is needed to organize forums devoted to industrial research and for students to participate in such forums. The major institutes, with their extensive contacts and experience, would be obvious facilitators of such activity.

A recent survey suggests that mathematics still does not fully exploit the employment opportunities offered by finance and high tech industries. The survey produced the following pattern of student employment. Of 355 PhDs graduating between 1992 and 1997, 190 ended up with academic jobs (96 tenure track, 41 PDFs, 44 limited term and 9 college appointments) and 101 in industry (29 in finance, 49 in high tech industry, 7 government and 16 other), 15 are retraining, and 49 are unknown. A very large percentage of those in the unknown category are foreign students who have left Canada. The very favourable employment situation for mathematics in the academic world (this will be discussed in the funding section) suggests that it is not unrealistic to have a continuing pattern of over 50% of PhD graduates employed in the academic world. However, one would anticipate a growing number of students employed in the financial/industrial world. An improved infrastructure to handle industrial training will clearly facilitate this. Indeed, most of the students who have entered finance and industry are from a few major centers where some such infrastructure is already present.

The three major institutes will play an increasingly important role in working with the GSC grantees to enhance the education of students and to conduct outreach on a more widespread and systematic basis. The full impact on mathematical research and education of the growing links of the institutes with industry and finance is only beginning to be explored. This is clearly an area in which new initiatives by the institutes will be occurring in the next few years. Some of these initiatives are described below.

Section A.IV: Funding Needs and Proposals

Currently in Mathematics there are both strong funding pressures and funding challenges. We will present three type I proposals which are designed to address these concerns. Our proposals are presented starting with the one of highest priority.

New applicants and younger researchers. The need to provide adequate funding for new applicants to the GSC is the most pressing funding concern for Mathematics. Increasing budget pressures make this goal difficult to attain without a significant increase in the level of GSC funding. Funding pressures from new arrivals will be increasing in the future. NSERC projections for Faculty departures suggest that at least 150 new Mathematics grantees will enter the system during the period 1997--2003 (Table 18 of DD). On the other hand, the current top researchers will not be leaving the system during that period and their funding needs will not diminish. Indeed the argument in Proposal I.3 (below) is that these researchers are already underfunded. As an indication of the longevity of the top level of researchers, if one considers the top 35 grantees in Mathematics (i.e., those with grants of at least twice the average size) then only one has taken early retirement while one other will reach retirement age before 2003.

Over the past years, Mathematics has given priority to their young promising researchers. Figure 13 and Table A22 of DD shows that young mathematicians are among the most successful of all the young scientists and engineers applying for first grants. Even during difficult times this commitment to new applicants was maintained; and the current GSC has declared the funding of new applicants to be a committee priority. This policy towards new applicants has been fruitful. Canadian mathematics includes a large number of younger researchers at the forefront of exciting emerging fields. Just to name a few, Darmon and Roy in Number theory; Behrend, Chen, Jeffrey, Khesin, Meinrenken and Nabutovsky in Geometry; Guan, Gui, Khalkhali, Nica, Putnam, Toth and Zworski in Analysis; Slade in Probability; Alama, Bronsard, Gannon, Hou, Keshin, Lewis, Loewen, Peirce, Quastel, Seco, Ward, Wu and Xu in Applied Analysis and N. Bergeron, Brown and Goddyn in Discrete Mathematics are all involved in first rate programs of research in emerging fields. Expanded support for our younger researchers is highly desirable. Involving this group in graduate supervision is a priority. More funding is required both for the basic support of students and for providing adequate training in the interdisciplinary and outreach areas where an increasing number of these younger researchers are found.

Proposal I.1. Additional funds ($5 K) for each of 150 projected *new* applicants in emerging fields and for 50 younger researchers in our GSCs during the period 1999--2003. This amounts to $1 M per year.

Training. There are two major challenges, with concomitant funding needs, associated with the training of highly qualified personnel in mathematics.

(i) There is a need to respond to a very positive employment environment. There has been an explosive growth of demand for mathematical talent from industry, finance, and business, combined with an exceptionally high need for Mathematicians in the academic world. (Table 15 of HQP ranks Mathematics second amongst all sciences for academic jobs). The Mathematics community has responded to this very favourable job market with a dramatic expansion of its graduate programs: Table 1 of HQP ranks Mathematics/Statistics as the highest of all sciences for the increase in Canadian enrollment at the graduate level during the ten year period 1986--1996 (77% for Masters and 96% for PhDs). Tables 3 and 4 of HQP confirm this pattern of sustained expansion. There is a need to substantially increase funding.

The estimates of employment prospects for mathematical scientists are astoundingly high (Tables 40 and 43 of HQP). The academic world offers very significant employment prospects for Mathematicians. Although in a similar position to other disciplines regarding replacement of Faculty during the next decade (Table 18 of HQP), the Faculty Demand/Supply index (Table 19 of HQP) is by far the highest for Mathematics, with a 55% score! This means that the recent steep increase in the number of doctoral degrees granted in Mathematics is still not enough to adequately deal with the coming Faculty departures. The pressure for the training of additional students in Mathematics will be very high during the next five years.

(ii) The other main challenge in the training of future graduate students is that posed by the rapidly evolving interdisciplinary aspects of mathematics (as discussed in the HQP section). An increasing number of younger researchers are being hired in these areas. Significantly more funds are needed to provide a broad and relevant training for their students. Among the additional needs of such students are: (1) complementary training at both the Masters and PhD levels in financial and technology-based industries; (2) collaboration with the major institutes (and affiliated graduate schools such as the IAM in Vancouver and the ISM in Quebec) to provide an efficient and flexible one-year program of training in situ at the three major institutes in collaboration with the private sector.

Proposal I.2. Additional funding to cope both with the ongoing increase of enrollment in our graduate programs, and with the new needs arising from the interdisciplinary aspects of the training. Our GSCs need increased funding of at least $5 K for each of the 80 additional graduate students projected in Table 4 of HQP for the period under review. This amounts to $400 K per year.

Top researchers. There is a clear need for additional funding of the top researchers in mathematics. There are 80 senior mathematicians with funding over 1.8 times the average grant. They all have international stature, and the top 35 are world leaders in mainstream fields with major international impact. These 80 people have had, on average, 2.1 Masters degrees and 1.9 PhDs degrees conferred under their supervision since 1992, in addition to 4 years of PDF support. This compares very favourably with that for top mathematical researchers in the USA. The top 30 US mathematics departments (1995 NRC rankings) have graduated, on average, 0.3 Master's and one PhD per tenured faculty member during the past three years.

This activity is being carried out with a level of funding lower than almost any other discipline. The average NSERC grant of the top 35 internationally acclaimed Canadian researchers is less than $35 K! Moreover, mathematicians depend on Research Grants funding in a way which no other discipline does. 90% of their NSERC funding comes from Research Grants funding. (See Table 14 of DD.) Mathematicians feel particular pressure in funding PDFs. Although PDFs play a very important role in mathematical research, even the larger grants in mathematics are insufficient to support them---usually cooperative ventures or outside funding is required. The above average of 4 postdocs is testimony to the outstanding success of mathematicians in accessing such funding. The cancellation of noncore programs has removed the major source of outside funding and forced greater reliance on Research Grants. As the Review of Canadian Mathematics commented *the systematic way in which mathematicians feel compelled to apply to non-mathematical GSCs as soon as they do some interdisciplinary work proves that the basic level of funding for Mathematics projects is lower.* Mathematics is one of the few disciplines to experience a significant decline in the number of grantees over the past 5 years. (See Table A1 of DD.) This reflects the attrition mentioned by the Review as well as the increased selectivity being exercised by the Mathematics GSCs. The role played by our top Canadian mathematicians is crucial to Canadian science and the above situation should be remedied.

Proposal I.3. An adjustment in funding for our top researchers is urgently needed. There must be an increased funding of $5 K for each of the 80 top researchers and an additional $5 K for the top 35 researchers with major international impact. This amounts to $575 K per year.

The following table summarizes these funding requests.

1.	New applicants and younger researchers	$1,000 K
2.	Training	$400 K
3.	Top researchers	$575 K
	Total	$1,975 K

Table 1. Budget Request for Mathematics GSCs 336 & 337

Section A.V: Conclusion

Over the past three years the Canadian mathematics community has undergone a fundamental re-evaluation. That process has confirmed a number of strengths and also revealed important areas of future growth. We have documented compelling arguments for both the impact and potential of Canadian mathematics and of our urgent need for more adequate funding. The Reallocation exercise has been established to insure a fair distribution of funds among the various GSCs according to the merits, dynamics, and proposals of each discipline. We request the Reallocation Committee to recognize the needs of Mathematics and to respond vigorously.

Part B: The Research Institutes for the Mathematical Sciences

Section B.I: The National Research Network

The past three years has seen the emergence of a National Research Network in the Mathematical Sciences. It is centred on three large institutes in Montreal, Toronto and Western Canada, but also embraces a number of other mathematical groups across the country. It is the outcome of intensive planning and work by mathematical scientists across the country and has received strong endorsement from the community on at least six occasions during 1995--1997. The Review of Canadian Mathematics was struck by the clarity and purpose with which Canadian mathematicians have pursued this goal and commented that they *have developed a striking sense of common destiny*.

The CENTRE DE RECHERCHES MATHEMATIQUES (CRM) has a thirty year history as a centre for mathematical research and has built an enviable international reputation. It currently receives $800 K per year in NSERC funding and has a total budget of $2 M. It is based at the Universite de Montreal and is affiliated with the other three Montreal universities and the Institut des sciences mathematiques (ISM), responsible mainly for graduate study coordination. Over the last two decades, the CRM developed a national institute model for research in the mathematical sciences in Canada, funding and organizing a variety of scientific activities throughout the country. The CRM has a tradition of fostering interdisciplinary projects and has an impressive record for effective and innovative liaison activities with industry and finance.

The FIELDS INSTITUTE FOR RESEARCH IN MATHEMATICAL SCIENCES (FI) was created at the beginning of this decade and has been situated on the University of Toronto campus since 1995. The FI has four major sponsoring universities in southern Ontario (McMaster, Toronto, Waterloo, York) and another eight affiliated institutions across the country. It receives $800K per year in NSERC funding and has a total budget of $2M. In its short period of existence, it has already established an important national role and increasingly serves as a focal point for the mathematical community. Its international profile was noted by the The Review of Canadian Mathematics which commented that *The Fields Institute has established itself on the world stage* and that *The Fields Institute is a major success story for science in Canada.*

The PACIFIC INSTITUTE FOR THE MATHEMATICAL SCIENCES (PIms) was created over the past three years by many of the leading scientists in Western Canada and held its inaugural events at the Universities of Calgary and Victoria in the fall of 1996. It reflects the emergence in the last twenty years of an internationally renowned community in the mathematical sciences in Western Canada. PIms is a cooperative venture involving the five large BC and Alberta Universities (Alberta, Calgary, SFU, UBC, Victoria). It is committed to the use of new technologies to link the research groups in Western Canada. It currently receives $200K from the University--Industry component of NSERC's Research Partnerships Program (RPP) for the development of outreach activities and has a total budget of $1M. The NSERC RPP funding will terminate in 1999.

These institutes have the mandate and ability to carry on a broad range of activities in mathematics, and more generally, the mathematical sciences. These include:

- large scale research programs of international importance. There is a strong emphasis on innovation: emerging areas, areas at the interface of different branches of mathematics, areas that cross traditional discipline boundaries.
- outreach initiatives that build partnerships among the mathematical sciences, other disciplines and the business/industrial sector. The emphasis is on fostering research that will lead to innovative applications of mathematics in business and industry.
- comprehensive training both at the graduate and postdoctoral level providing young mathematicians with exposure to significant and innovative research of the type described above.
- promotion of the mathematical sciences to the public and to serve as a resource for mathematical educators at all levels.

National institutes for research in the mathematical sciences are now commonplace (e.g., Isaac Newton Institute in the U.K., two Max Planck Institutes in Germany, RIMS in Japan, MSRI and IMA in the US). Such institutes are deemed essential to remain competitive internationally. In the Canadian context a network of institutes is by far the most effective configuration to achieve the above stated national mandates. The large distances in our country and its regionally concentrated population means that the present three institutes, as well as regional associations, are needed to ensure the effective participation of all Canadian mathematicians. It is important to realize that research institutes have a major impact on, and major reliance upon, scientific activity in their region. They both stimulate and depend upon strong local academic communities. Moreover, the outreach of the institutes is focused upon local business and industry. And such regional networks of industrial partnerships increasingly serve as catalysts for innovative interdisciplinary graduate programs. Regional presences are needed to achieve national goals.

The Directors of the three institutes are developing a long-term strategy for the National Network to respond to their national mandates. The three Institutes will jointly plan major theme programs and a joint scientific panel will be established to review offsite funding applications. For example, FI and Pims will jointly run a full year program in Graph Theory and Combinatorial Optimization for 1999--2000. Finally, in spite of their own budgetary limitations, the CRM and FI have together committed $100K of bridging support for scientific programs at PIms for 1998--99.

To fulfill their role as the basis of a National Network, the three Institutes also wish to expand their role of encouraging and supporting groups other than those directly affiliated with the Institutes. There is already a tradition of joint funding and organizing of smaller meetings and conferences between the FI and CRM; and the Vancouver Meeting in Probability (Aug. 19--28, 1997) was the first meeting jointly sponsored by all three Institutes. One particular function of the joint scientific panel dealing with off site funding applications is to review and help fund high level scientific workshops, summer schools and outreach activities initiated by regional associations such as the Atlantic Association for Research in Mathematical Sciences (AARMS) and Central Canadian Association for Research in Mathematical Sciences (CCARMS) and by the professional societies (CMS, CAMS and SSC).

This configuration of three research institutes creates an ideal framework for collaborative research and the development of highly qualified personnel in the mathematical sciences in Canada. It combines regional networks of academic and industrial partners with linkage at the national level for the coordination of scientific program planning, training initiatives and outreach.

Section B.II: Research Programs

The CRM has sponsored full scale thematic terms or years on specific areas in the mathematical sciences since 1990, while The Fields Institute adopted these thematic programs from its very beginning in 1992. Both the FI and CRM have the staff as well as the computing and office facilities for hosting over 50 scientists for long periods of intense study. These periods not only lead to significant progress in a particular field, but also provide an opportunity for graduate students and young researchers to learn from and interact with the leading scientists in the world. The distributed nature of PIms lends itself to shorter thematic periods distributed among its sites. Beginning with the program in Probability and its Applications in the summer of 1997 (cosponsored by FI and CRM), it has started to host Thematic Summers in areas which complement activities at the institutes.

- A striking example of the long-term value of thematic programs was the year-long research program on Operator Algebras and Applications organized by George Elliott at FI in 1994--95. This program brought together senior scientists, 37 postdoctoral fellows and 63 graduate students for extended periods of intensive research. This led to significant research advances in several aspects of operator algebras which were documented in four volumes of the Fields Institute Communications. The two full year graduate courses, lecture series and seminars led to the publication of six monographs. The program was so successful that it was extended to 1995--1996 with external funding to support five doctoral students and seven postdoctoral fellows. This program has had a lasting impact on Canadian mathematics and on the careers of the many young mathematicians who participated.
- An example of the breadth and interdisciplinary impact made possible by these thematic programs is the Theme Year on Numerical and Applied Analysis held at the CRM in 1995--1996. This program began with a Summer School on Boundaries, Interfaces and Transition, followed in the fall by a range of workshops and minicourses in Fluid Mechanics. The second term offered programs on wavelets and their applications, artificial neural nets, mathematics of finance, and modern special function theory. Besides pure and applied mathematicians, the events also gathered computers scientists working in the film industry, engineers concerned with airplane modeling or image processing through telecommunications, bankers and economists. The activities of this interdisciplinary program were recorded in 11 CRM volumes published by the AMS, including a best-selling book on financial mathematics.
- Table 2 demonstrates the broad spectrum of the mathematical sciences as well as the variety of applications to other disciplines covered by these programs. Programs of this magnitude allow one to transcend discipline boundaries. For example, as part of the Probability year at FI in 98--99, a session on Stochastic Models in Biology will include probabilists, geneticists and ecologists and will lead into the Thematic Summer at PIms in Mathematical Biology. Similarly, the 97--98 CRM year in Statistics will bring statisticians together with geneticists, epidemiologists and neuroscientists.

Year	CRM	Fields Institute	PIms
95--96	Applied and Numerical Analysis	Long range aperiodic order Homotopy Theory	
96--97	Group Theory and Combinatorics	Algebraic Model Theory Singularity Theory and Geometry	Probability
97--98	Statistics	Microlocal Analysis Complexity Theory and Algorithms	Mathematical Economics and Mathematical Finance
98--99	Number Theory and Algebraic Geometry	Probability and Applications	Mathematical Biology
99--00	Mathematical Physics	Graph Theory and Combinatorial Optimization	Graph Theory and Computational Geometry

Table 2. Institute Thematic Programs

Activities of this scope and impact in the mathematical sciences are not possible in this country without the infrastructure offered by the Institutes. In particular, the interdisciplinary programs are much harder to organize in departmentalized university environments.

Miniprograms, Workshops and Conferences. The Network plans to organize a larger number of targeted programs of short duration (1--3 months). These focus on newly emerging areas of mathematical research, on fields at the interface with other disciplines, or on problems emerging in the private sector. The smaller scale allows such meetings to be planned quickly in response to a new development or application. Typical examples coming from Canadian industry would include the many mathematical challenges emerging from the field of broadband communications---for example, optimum frequency allocation, ATM network traffic, and the mathematics of soliton transmission systems. Miniprograms of this type are currently planned at all three institutes.

The Mathematics GSCs used 20% of the now discontinued conference program compared to about 5% of the NSERC Research Grants Budget. This disproportionate use reflects the central importance of conferences and workshops in mathematical research. Support of such research interactions has landed on the shoulders of the CRM and FI, and more recently PIms, adding to the budget pressure. The three Institutes will work together to evaluate and support 'offsite' conferences and workshops.

Training. The importance of training a new generation of mathematical scientists is underlined by the NSERC predictive data in Table 40 of HQP which indicates a growth rate of 21.3% for mathematicians and statisticians for the period 1996--2001 (exceeded only by computer science, aerospace and computer engineers). The training strategy of the institutes has two main components.

First, the financial support of graduate students and postdoctoral fellows forms an essential component of both the thematic programs and the miniprograms. These programs offer young researchers an ideal opportunity, not only to present their research, but also to interact with the leaders in their field, and to be an integral part of the institute research life. For example, during 1996--97 over 200 graduate students and 100 postdoctoral fellows participated in the various activities of the Fields Institute. In addition, summer schools for graduate students on carefully selected topics are regularly supported by the Institutes. For example, since 1990 the CRM has organized a summer school (held in Banff since 1994) which is the first activity of the CRM theme year and is designed to prepare students for some of the events that will follow during the year. In 1997, AARMS (with CRM and FI support) held a summer program in differential geometry, the first of a series of summer schools to be held (funding permitted) in Atlantic Canada linked to thematic programs at the three Institutes. To increase participation in these programs, the Network is planning a new graduate mobility program to pay for travel and local expenses of the best graduate students.

Secondly, the new links with business and industry teach Canadian graduate students to view non-academic careers as exciting and challenging possibilities. They also provide valuable feedback for planning graduate programs at the affiliated departments. The CRM, FI and PIms are cosponsoring postdoctoral fellows with such companies as Lockheed-Martin, BC Transit, Siemens, Maple, Amber Computer systems, Canadian Cable Labs and R.I.T.A. Labs in the west and Nortel, Ad Opt and Cognologic, Bombardier, Pratt & Whitney, Environment Canada, GE Canada, Lockheed--Martin, Atlantic Nuclear Services and Hydro Quebec in the east.

Section B.III: Outreach to the Private Sector

All three Institutes have committed significant resources to, and succeeded in, establishing links with business and industry. Outreach of this kind requires a regional Institute with a local industrial base (Toronto, Montreal and Vancouver/Calgary/Edmonton), access to a large scientific base and resources to bring the two together. The rewards of such connections are many: new applications of mathematics, the feedback to novel areas of mathematical research, and new job opportunities for graduate students in mathematics.

Most of the connections to date have been with the financial world and with technology-based industries.

These are areas in which widespread innovative applications of mathematical expertise are occurring at an accelerating pace. They are also areas of major future growth. Technology-based industries are the key to job creation and to success in the global economy. Recent statistics (1997 Ontario Budget Paper E: The R&D Opportunity) show that roughly two out of every three jobs created in the past ten years were in industries in the telecommunications, financial services, aircraft, business machines, pharmaceuticals and medicine, computing, and engineering and scientific services sectors. These industries are already indicating that their demand over the next decade for mathematicians involved in R&D will increase dramatically. In the past three years a number of very successful outreach projects have been developed by the mathematical institutes. As representative, and seminal, examples we cite:

- The CRM has a history of outreach to business and industry which this past year culminated in the formation of The Network for Computing and Mathematical Modelling (NCM_2). This Montreal based consortium for research in industrial mathematics arose under CRM's leadership. It currently has three main research areas: risk management; information processing, imaging, parallel computing; transportation and telecommunication. NSERC has funded it for the period 1997--2002 through the Research Networks component of the Research Partnership Program. For every dollar of NSERC support, NCM_2 projects have received nearly two dollars from close to 20 non-academic industries, financial organizations and government agencies. This provides a striking demonstration of the potential for partnerships between the Institutes and Canadian industrial R&D.
- Since 1995, the Fields Institute Seminars in Financial Mathematics have had a major impact on the vibrant Canadian financial services industry. These monthly interdisciplinary seminars involve mathematical, statistical and computational issues and attract an average audience of over 100 participants from over ten financial institutions, including all major banks, and six universities. They provide an exceptional opportunity for graduate students and postdoctoral fellows, and have resulted in industrial employment for a growing number. A second series of Industrial Mathematics Seminars has now been initiated which will focus on industrial sectors such as information technology, telecommunications and aerospace.
- In August 1997, PIms organized a highly successful 'Industrial Problem Solving Workshop'. Eighty mathematical scientists from all five PIms campuses, the US and Britain worked collectively for a full week on mathematical problems proposed by MacMillan--Bloedel, Powertech, Petro-Canada, Kinetic Science Ltd., and the BC Cancer Agency. The fact that PIms could draw on expertise from a broad scientific base was essential for the high rate of success on the six proposed problems. More importantly, ongoing links have been established with a number of companies. This activity has now grown into an Annual Forum on Industrial Mathematics beginning next spring. A week long Graduate Industrial Modeling Workshop at SFU will train graduate students in the modern methods of applied mathematics so that they can actively participate in the Second Industrial Problem Solving Workshop to be held at U. Calgary in the following week.

Three PIms industrial facilitators in Vancouver, Calgary and Edmonton are working to connect mathematicians, computer scientists and statisticians with scientific projects in the public and private sector. The development of such connections with the private sector takes considerable effort and would not have happened without the resources offered by the Institutes. The institutes are currently diversifying their industrial partners, and funding is required to support the development and establishment of these links.

Section B.IV: Type II Proposal for the Consolidation of a National Research Network

In this section we present in succinct form the proposal for the consolidation of the National Network of Institutes which is the result of three years of planning and development. There is a clear national consensus on the basic objectives of the proposal:

- To complete the Western node of the National Network by insuring base funding for PIms.

- To sustain CRM and FI as world class research institutes.
- To fund the new national initiatives and innovative projects of the the network of the three institutes.

The Review of Canadian Mathematics strongly endorsed this strategy and recommended that *NSERC should find a way to support all three institutes* (*CRM, FI and PIms*) *by proposing a scheme of funding which encourages a national vision though collaboration and complementary activities.*

Based on the above recommendation of the Review of Canadian Mathematics, NSERC decided that the funding competition for the period 1999--2003 will be open to the three institutes CRM, FI and PIms with the precise allocation of funds among the three institutes to be decided by an independent international panel. This is the rationale behind the present request for new Type II funding for the Network of \$1.2M which will bring the total funding for the three institutes to a level of \$2.64M.

Base funding for PIms. Since its creation in 1995, PIms has unified the community of mathematical scientists in Western Canada, and engendered a many-fold increase in organized activity in mathematical research, industrial collaboration, education and public outreach. During its first year PIms sponsored 28 scientific activities of high quality, at all five sites and elsewhere, involving over a thousand scientists and students. PIms provided postdoctoral opportunities, which did not previously exist, for twenty highly qualified young mathematical scientists. Its distributed nature has proven to be effective in creating a synergy involving hundreds of scientists and scholars in Western Canada, who had previously worked in relative isolation. Its broad regional base has allowed it to develop a productive dialogue among university scientists, local industry and the governments in Alberta and BC.

- *An adaptive structure:* PIms has developed an innovative distributed structure sharing resources among the five university sites together with a flexible program structure and use of technology such as video-conferencing, interactive web-objects and other on-line research and computational tools. An NSERC 1996 site visit committee consisting of a top international team of scientists and engineers wrote in its report *The Site Visit Team is convinced that given the goals, the demography of the two Western Provinces, and the existence of mathematical strengths at five different universities, the distributed institute concept---a network---is likely to be much more effective than would the establishment of a single-site institute.* As an example, the already cited 'Industrial Problem Solving Workshop' could not have been organized by PIms without the combined resources of all five member universities.
- *Innovative programs:* PIms has developed innovative programs such as shorter and varied thematic periods, proactive outreach to other disciplines and the development, in collaboration with the private sector, of new media for doing and communicating mathematics. PIms was carefully scrutinized by NSERC before it received its interim funding. The above cited international panel also wrote in its report *To summarize, the quality and nature of the scientific programs proposed by PIms is excellent, based on great scientific strength and appropriate structure.*
- *Links to Pacific Rim countries:* PIms provides the Canadian mathematical sciences community with natural scientific links with Pacific Rim countries. For example, the First Pacific Rim Mathematics Conference will be held in Hong Kong in January 98 and is jointly sponsored by PIms, the Centre for Mathematical Sciences at the City U. of Hong Kong, and the Institute for Mathematics in Taiwan. In July 98, PIms will be hosting the first Pacific Rim Geometry Conference to be held in North America. An ongoing agreement on summer schools and workshops in mathematical physics between the Asia--Pacific Centre in Korea and PIms has been recently arranged.
- *Involvement of Western provincial governments and industry:* NSERC support for PIms provides an important new means of leveraging of financial support for mathematics. PIms is currently supporting 7 industrial PDFs with 2/3 of the salaries coming from the private sector. NSERC support for PIms was instrumental in securing funding of 50 cents on the federal dollar from the BC provincial government who used the above cited NSERC peer-review of PIms as the basis for their decision. NSERC support is currently augmented by Western Canadian universities whose support now totals \$400K in cash in

addition to the infrastructure, and is an essential ingredient of the pending application for funding to the Alberta Science Research Authority.

Proposal II.1. \$800 K of Type II funds for the support of PIms programs. This request takes into account the expected financial constraints on the institute's envelope and is the minimal level of funding required from NSERC to achieve the objectives presented in this document. See Table 3 below.

1.	Thematic programs	\$200 K
2.	Miniprograms/workshops/new initiatives/Network programs	\$100 K
3.	Scientific personnel/Training	\$200 K
4.	Technology-based mathematical sciences	\$100 K
5.	Industrial outreach activities	\$200 K
	Total	\$800 K

Table 3. Budget Request for PIms

Notes. 1. Thematic programs in BC and Alberta with emphasis on emerging areas and those at the interface of different disciplines: Mathematical Economics and Finance 1998, Mathematical Biology 1999, Computational Geometry 2000, Scientific Computing 2001. In the planning stages are also thematic programs in Knot Theory 2000 and Partial Differential Equations 2001.

2. Miniprograms, workshops, new initiatives, international collaborations as well as PIms contribution to the National Network programs cited below.

3. Seed money for senior visitors, postdoctoral fellows and graduate students connected to PIms activities, to be matched by university and provincial government sources. PIms is currently supporting 13 PDFs with 2/3 of the salaries coming from non-PIms sources.

4. PIms research program on the application of new technologies for research and training in the mathematical sciences: As an illustration, we cite the Polymath Development Project at SFU for developing on-line Java-based resources for the mathematical community, the Fluid dynamics Laboratory at U. Alberta for performing numerical simulations and other mathematical experiments dealing with environmental and industrial problems, and the Sunsite at UBC which is being developed in collaboration with Sun Microsystems. Potential matching funds are of the order of \$700 K.

5. This will essentially replace the current interim funding from NSERC for PIms industrial outreach activities: Industrial facilitators in BC and Alberta, PIms Annual Industrial Forum, Industrial PDFs. Again, this is seed money for a much larger envelope from mostly industrial sources.

Funding for CRM and FI: *New Initiatives and National Network programs.* The CRM and FI have an enviable track record in supporting high quality research programs. During 1996--97 the programs of the CRM and FI each attracted over 1500 mathematical scientists from over 50 countries, and had a major impact on the mathematical sciences research community in Canada. Both institutes have been reviewed by a number of national and international bodies. They have been highly evaluated for the quality of their scientific activities and their cost effectiveness. Both are presently funded at a minimal level well below funding of institutes in other countries. Although there are minor differences from year-to-year and between the institutes, the NSERC grant of \$800 K at both CRM and FI is typically budgeted as shown.

Thematic Programs	\$400 K
Senior scientists \$60 K, PDFs \$100 K, graduate students \$50 K, workshops \$160 K, graduate courses/seminars \$30 K	
General and national programs	\$180 K
General and offsite workshops \$130 K, industrial programs \$50 K	
Scientific support	\$220 K
Scientific program coordinators \$100 K, computer support \$60 K, administration expenses for scientific programs \$60 K	
Total	\$800 K

Table 4. Typical CRM and FI Disbursement of NSERC Funds

The CRM and FI have leveraged these research funds to obtain $1.2M each in additional support from local sources, mainly the provincial governments of Quebec and Ontario and the universities. This additional funding contributes most of the basic infrastructures and supports publishing programs, graduate students and PDFs, industrial outreach and educational programs. Consequently, every additional dollar the institutes receive now goes into direct scientific support. The current resources of the institutes are stretched to the limit in providing this range of activities and new funding is essential to maintain the world class level of these programs while undertaking the new national initiatives. In particular exceptional budgetary pressures are being felt in the following areas.

- *Interdisciplinary and National Programs:* There is a growing need for interdisciplinary miniprograms, and the institutes have an increased responsibility for the funding of off-site activities as well as collaborative initiatives with national and regional organizations (CMS, CAMS, SSC, AARMS, CCARMS). This requires new base funding.
- *Training and Scientific Personnel:* Current budgets do not allow for the release of a significant number of scientists for long term participation in the thematic programs. The impact of a few such visitors on a program is enormous. Also the current budget for PDFs is heavily leveraged; nevertheless it only represents the equivalent of three fulltime positions. Finally, the new national summer school and graduate student mobility program require additional funding.
- *An Emerging Industrial Vocation:* This is an area of important future growth in which the institutes already have a substantial track record. New funding is needed for these initiatives, in particular for industrial PDFs and to provide seed money to develop new industrial partnerships.

Proposal II.2. $400K of Type II funds for the support of new FI and CRM initiatives and National Network activities as outlined below.

1. Miniprograms/Network Programs	$120K
2. Training/Scientific Personnel	$140K
3. Industrial Outreach Activities	$140K
Total	$400K

Table 5. Budget Request for CRM and FI

Section B.V: Conclusion

Over the last six-year period, the CRM and FI have evolved into internationally recognized institutes in the mathematical sciences. This development has been achieved with annual NSERC support of $1.6M, leveraged by an additional $2.4M from provincial sources. In the past two-years, PIms has established itself as a highly innovative and successful institute through a major initiative of mathematical scientists in Western Canada with the financial support of their universities and provincial govenments, plus interim NSERC funding. Additional base funding is essential in order to capitalize on the important new leveraging opportunity provided by PIms and the exceptional potential for Canadian science created by the national network centered on the three institutes. The current funding including the 10% in the reallocation pool is barely adequate for two of these institutes to carry on a full range of activities. Without additional funding, the network of three institutes cannot effectively function and the impressive current level of regional funding could be jeopardized. Thus the mathematical sciences in Canada are at a crucial turning point. We are requesting the addition of $1.2M to the Institute envelope in order to fully realize the potential for Canadian science provided by PIms, to continue the development of CRM and FI as world class research institutes, and to enable the three institutes to collectively support mathematical research, training and outreach throughout the country.