Report of the Thirty Eighth
Canadian Mathematical Olympiad
2006

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The Canadian Mathematical Olympiad (CMO) is an annual national mathematics competition sponsored by the Canadian Mathematical Society (CMS) and is administered by the Canadian Mathematical Olympiad Committee (CMO Committee), a sub-committee of the Mathematical Competitions Committee. The CMO was established in 1969 to provide an opportunity for students who performed well in various provincial mathematics competitions to compete at a national level. It also serves as preparation for those Canadian students competing at the International Mathematical Olympiad (IMO).

Students qualify to write the CMO by earning a sufficiently high score on the Canadian Open Mathematical Challenge (COMC). Students may also be nominated to write the CMO by a provincial coordinator.

The Society is grateful for support from the **Sun Life Assurance Company of Canada** as the Major Sponsor of the 2006 Canadian Mathematical Olympiad and the other sponsors which include: the Ministry of Education of Ontario; the Ministry of Education of Quebec; Alberta Learning; the Department of Education, New Brunswick; the Department of Education, Newfoundland and Labrador; the Department of Education, the Northwest Territories; the Department of Education of Saskatchewan; the Department of Mathematics and Statistics, University of New Brunswick at Fredericton; the Centre for Education in Mathematics and Computing, University of Waterloo; the Department of Mathematics and Statistics, University of Ottawa; the Department of Mathematics, University of Toronto; the Department of Mathematics, University of British Columbia; Nelson Thompson Learning; John Wiley and Sons Canada Ltd.; A.K. Peters and Maplesoft.

My thanks go to Robert Bilinski of the Collège Montmorency for nominating students from the Province of Québec for the CMO, Peter Minev of the University of Alberta for nominating students form the Province of Alberta and to Rob Craigen of the University of Manitoba for checking for eligible students in his province.

I am very grateful to the CMO Committee members who helped compose and/or mark the examination: Robert Barrington Leigh (University of Toronto), Man-Duen Choi (University of Toronto), Chris Fisher (University of Regina), Richard Hoshino (Dalhousie University), Roger Mong (University of Toronto), Igor Poliakov (York University), Felix Recio (University of Toronto), Naoki Sato (San Diego, CA), Jacob Tsimerman (University of Toronto), Terry Visentin (University of Winnipeg) and Ed Wang (Wilfrid Laurier University). Thanks to Tom Griffiths of London, ON for serving as validator and to Joseph Khoury for translating the paper into French. I acknowledge the support of the CMO Competitions Committee chaired by George Bluman (University of British Columbia), and am indebted for the hard work done at CMS headquarters by Nathalie Blanchard and the Executive Director, Graham Wright, whose continued commitment is a vital ingredient of the success of the CMO.

Ed Barbeau, Chair
Canadian Mathematical Olympiad Committee

The 38th (2006) Canadian Mathematical Olympiad was held on Wednesday, March 29, 2006. A total of 79 students from 55 schools (52 in Canada, two in the US and one in Singapore) wrote the paper; eight Canadian provinces were represented. The number of contestants from each province was as follows:

BC(14) AB(8) SK(1) MB(1) ON(40) QC(6) NB(2) NS(1)

The 2006 CMO consisted of five questions, and the maximum score was 33. The contestants were grouped into four divisions according to their scores as follows:

Division	Range of Scores	No. of Students
I	$18 \leq m < 28$	8
II	$14 \leq m < 17$	16
III	$9 \leq m < 13$	21
IV	$1 \leq m < 8$	34

FIRST PRIZE — Sun Life Financial Cup — $2000

Dong Uk (David) Rhee
McNally School, Edmonton, Alberta

SECOND PRIZE — $1500
Yufei Zhao
Don Mills Collegiate Institute

THIRD PRIZE — $1000
Shawn Eastwood
Canadian International School

HONOURABLE MENTIONS — $500

Alan Guo
O'Neill Collegiate and Vocational Institute
Oshawa, ON

Kent Huynh
University of Toronto Schools
Toronto, ON

Viktoriya Krakovna
Vaughan Road Academy
Toronto, ON

Alex Remerov
Waterloo Collegiate Institute
Waterloo, ON

Thomas Tang
A.Y. Jackson Secondary School
North York, ON

Division 2

$14 \leq m < 17$

Farzin Barekat	Sutherland Secondary School	BC
Dimitri Dziabenko	Don Mills Collegiate Institute	ON
Xiaoshi Huang	Sir Winston Churchill High School	AB
Lei Jia	Waterloo Collegiate Institute	ON
Steven Karp	Lord Byng Secondary School	BC
Adrian Keet	Westmount Charter School	AB
Andy Kong	Vincent Massey Secondary School	ON
William Ma	Waterloo Collegiate Institute	ON
Johnson Mo	St. John's School	BC
Jeffrey Mo	William Aberhart High School	AB
Jennifer Park	Bluevale Collegiate Institute	ON
Yongho Park	Richmond Hill High School	ON
Richard Peng	Vaughan Road Academy	ON
Chen Sun	Tom Griffiths Home School	ON
Alex Wice	Leaside High School	ON
Alex Xu	Indus Center for Academic Excellence	MI

Division 3

$9 \leq m < 13$

Nicolas Berube	College de Bois-de-Boulogne	QC
Boris Braverman	Sir Winston Churchill High School	AB
Lin Fei	Don Mills Collegiate Institute	ON
Ioan Filip	Marianopolis College	QC
William (Jiening) Fu	A.Y. Jackson Secondary School	ON
Jimmy He	Seaquam Secondary School	BC
Jiayang Jiang	A.Y. Jackson Secondary School	ON
Fang Lu	Glebe Collegiate Institute	ON
Frank Meng	Burnaby South Secondary School	BC
Clare Park	St. Theresa of Lisieux Collegiate H.S.	ON
Luke Schaeffer	Centennial C. & V. I.	ON
Jonathan Schneider	University of Toronto Schools	ON
Peng Shi	Sir John A. Macdonald Collegiate Institute	ON
Tony Wan	Stephen Leacock Collegiate Institute	ON
Ze Wang	Colonel By Secondary School	ON
Chen Xi	Harry Ainly High School	AB
Bobby Xiao	Walter Murray Collegiate Institute	SK
Hao Yan	Jarvis Collegiate Institute	ON
Yiyi Yang	Western Canada High School	AB
Allen Zhang	St. George's School	BC
John Zhou	Indus Center for Academic Excellence	MI

Division 4

$1 \leq m < 8$

Sunil Agarwal	Indus Center for Academic Excellence	MI
Vivek Behera	Indus Center for Academic Excellence	MI
Eunse Chang	Don Mills Collegiate Institute	ON
Harry Chang	A.B. Lucas Secondary School	ON
Derek Chiu	Crescent School	ON
Bo Hong Deng	Jarvis Collegiate Institute	ON
Julia Evans	John Abbott College	QC
Joe Kileel	Fredericton High School	NB
Ben Krause	St. George's School	BC
Michael Lee	The Woodlands School	ON
Robert Legassicke	Dover Bay Secondary School	BC
Stanley Lei	York Mills Collegiate Institute	ON
Scott (Yi-Heng) Lin	Moscrop Secondary School	BC
Sunny Liu	Sir Winston Churchill Secondary School	BC
Ethan Macaulay	The Halifax Grammar School	NS
Yale Mao	The Woodlands School	ON
P Alexandre Menard	College de Bois-de-Boulogne	QC
Yuchen Mu	St. John's-Ravenscourt School	MB
Jeremy Pham	The Advance Academy of Georgia	ON
Silviu Pitis	Don Mills Collegiate Institute	ON
Bruno Savard	Cegep St-Jean-sur-Richelieu	QC
Danny Shi	Windermere Secondary School	BC
Sarah Sun	Holy Trinity Academy	AB
Peter Sun	Sir Winston Churchill Secondary School	BC
Shirley Wu	Sir John A. Macdonald Collegiate Institute	ON
Lei Wu	Vincent Massey Secondary School	ON
Thomas Wu	Sir Winston Churchill Secondary School	BC
Kevin Xiong	Don Mills Collegiate Institute	ON
Vick Yao	Vincent Massey Secondary School	ON
Wei Zhong Ye	Fredericton High School	NB
Alan Ye	University Hill Secondary School	BC
Boyang Zhang	The Woodlands School	ON
Qiyu Zhu	A.Y. Jackson Secondary School	ON
Chenglong Zou	John Abbott College	QC

38th Canadian Mathematical Olympiad

Wednesday, March 29, 2006

1. Let $f(n,k)$ be the number of ways of distributing k candies to n children so that each child receives at most 2 candies. For example, if $n=3$, then $f(3,7)=0$, $f(3,6)=1$ and $f(3,4)=6$.

Determine the value of

$$f(2006,1)+f(2006,4)+f(2006,7)+\cdots+f(2006,1000)+f(2006,1003)\ .$$

2. Let ABC be an acute-angled triangle. Inscribe a rectangle $DEFG$ in this triangle so that D is on AB, E is on AC and both F and G are on BC. Describe the locus of (*i.e.*, the curve occupied by) the intersections of the diagonals of all possible rectangles $DEFG$.

3. In a rectangular array of nonnegative real numbers with m rows and n columns, each row and each column contains at least one positive element. Moreover, if a row and a column intersect in a positive element, then the sums of their elements are the same. Prove that $m=n$.

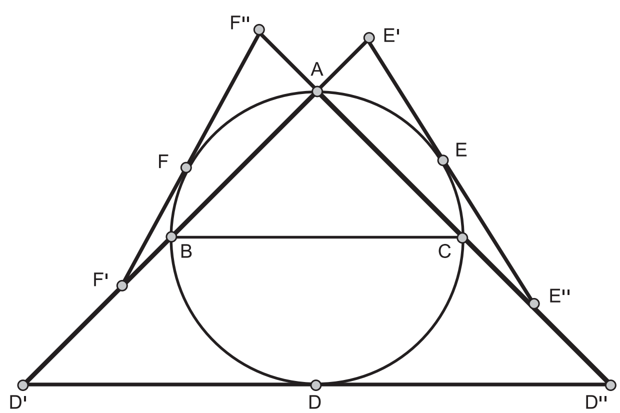
4. Consider a round-robin tournament with $2n+1$ teams, where each team plays each other team exactly once. We say that three teams X, Y and Z, form a *cycle triplet* if X beats Y, Y beats Z, and Z beats X. There are no ties.

(a) Determine the minimum number of cycle triplets possible.

(b) Determine the maximum number of cycle triplets possible.

5. The vertices of a right triangle ABC inscribed in a circle divide the circumference into three arcs. The right angle is at A, so that the opposite arc BC is a semicircle while arc AB and arc AC are supplementary. To each of the three arcs, we draw a tangent such that its point of tangency is the midpoint of that portion of the tangent intercepted by the extended lines AB and AC. More precisely, the point D on arc BC is the midpoint of the segment joining the points D' and D'' where the tangent at D intersects the extended lines AB and AC. Similarly for E on arc AC and F on arc AB.

Prove that triangle DEF is equilateral.



38th Canadian Mathematical Olympiad

Wednesday, March 29, 2006

Solutions to the 2006 CMO paper

1. Let $f(n,k)$ be the number of ways of distributing k candies to n children so that each child receives at most 2 candies. For example, if $n=3$, then $f(3,7)=0$, $f(3,6)=1$ and $f(3,4)=6$.

Determine the value of

$$f(2006,1)+f(2006,4)+f(2006,7)+\cdots+f(2006,1000)+f(2006,1003)\ .$$

Comment. Unfortunately, there was an error in the statement of this problem. It was intended that the sum should continue to $f(2006,4012)$.

Solution 1. The number of ways of distributing k candies to 2006 children is equal to the number of ways of distributing 0 to a particular child and k to the rest, plus the number of ways of distributing 1 to the particular child and $k-1$ to the rest, plus the number of ways of distributing 2 to the particular child and $k-2$ to the rest. Thus $f(2006,k)=f(2005,k)+f(2005,k-1)+f(2005,k-2)$, so that the required sum is

$$1+\sum_{k=1}^{1003} f(2005,k)\ .$$

In evaluating $f(n,k)$, suppose that there are r children who receive 2 candies; these r children can be chosen in $\binom{n}{r}$ ways. Then there are $k-2r$ candies from which at most one is given to each of $n-r$ children. Hence

$$f(n,k)=\sum_{r=0}^{\lfloor k/2\rfloor}\binom{n}{r}\binom{n-r}{k-2r}=\sum_{r=0}^{\infty}\binom{n}{r}\binom{n-r}{k-2r}\ ,$$

with $\binom{x}{y}=0$ when $x<y$ and when $y<0$. The answer is

$$\sum_{k=0}^{1003}\sum_{r=0}^{\infty}\binom{2005}{r}\binom{2005-r}{k-2r}=\sum_{r=0}^{\infty}\binom{2005}{r}\sum_{k=0}^{1003}\binom{2005-r}{k-2r}\ .$$

Solution 2. The desired number is the sum of the coefficients of the terms of degree not exceeding 1003 in the expansion of $(1+x+x^2)^{2005}$, which is equal to the coefficient of x^{1003} in the expansion of

$$\begin{aligned}(1+x+x^2)^{2005}(1+x+\cdots+x^{1003}) &= [(1-x^3)^{2005}(1-x)^{-2005}](1-x^{1004})(1-x)^{-1}\\ &= (1-x^3)^{2005}(1-x)^{-2006}-(1-x^3)^{2005}(1-x)^{-2006}x^{1004}\ .\end{aligned}$$

Since the degree of every term in the expansion of the second member on the right exceeds 1003, we are looking for the coefficient of x^{1003} in the expansion of the first member:

$$(1-x^3)^{2005}(1-x)^{-2006} = \sum_{i=0}^{2005}(-1)^i\binom{2005}{i}x^{3i}\sum_{j=0}^{\infty}(-1)^j\binom{-2006}{j}x^j$$

$$= \sum_{i=0}^{2005}\sum_{j=0}^{\infty}(-1)^i\binom{2005}{i}\binom{2005+j}{j}x^{3i+j}$$

$$= \sum_{k=0}^{\infty}\left(\sum_{i=1}^{2005}(-1)^i\binom{2005}{i}\binom{2005+k-3i}{2005}\right)x^k \ .$$

The desired number is

$$\sum_{i=1}^{334}(-1)^i\binom{2005}{i}\binom{3008-3i}{2005} = \sum_{i=1}^{334}(-1)^i\frac{(3008-3i)!}{i!(2005-i)!(1003-3i)!} \ .$$

(Note that $\binom{3008-3i}{2005} = 0$ when $i \geq 335$.)

2. Let ABC be an acute-angled triangle. Inscribe a rectangle $DEFG$ in this triangle so that D is on AB, E is on AC and both F and G are on BC. Describe the locus of (*i.e.*, the curve occupied by) the intersections of the diagonals of all possible rectangles $DEFG$.

Solution. The locus is the line segment joining the midpoint M of BC to the midpoint K of the altitude AH. Note that a segment DE with D on AB and E on AC determines an inscribed rectangle; the midpoint F of DE lies on the median AM, while the midpoint of the perpendicular from F to BC is the centre of the rectangle. This lies on the median MK of the triangle AMH.

Conversely, any point P on MK is the centre of a rectangle with base along BC whose height is double the distance from K to BC.

3. In a rectangular array of nonnegative real numbers with m rows and n columns, each row and each column contains at least one positive element. Moreover, if a row and a column intersect in a positive element, then the sums of their elements are the same. Prove that $m = n$.

Solution 1. Consider first the case where all the rows have the same positive sum s; this covers the particular situation in which $m = 1$. Then each column, sharing a positive element with some row, must also have the sum s. Then the sum of all the entries in the matrix is $ms = ns$, whence $m = n$.

We prove the general case by induction on m. The case $m = 1$ is already covered. Suppose that we have an $m \times n$ array not all of whose rows have the same sum. Let $r < m$ of the rows have the sum s, and each of the of the other rows have a different sum. Then every column sharing a positive entry with one of these rows must also have sum s, and these are the only columns with the sum s. Suppose there are c columns with sum s. The situation is essentially unchanged if we permute the rows and then the column so that the first r rows have the sum s and the first c columns have the sum s. Since all the entries of the first r rows not in the first c columns and in the first c columns not in the first r rows must be 0, we can partition the array into a $r \times c$ array in which all rows and columns have sum s and which satisfies the hypothesis of the problem, two rectangular arrays of zeros in the upper right and lower left and a rectangular $(m-r)\times(n-c)$ array in the lower right that satisfies the conditions of the problem. By the induction hypothesis, we see that $r = c$ and so $m = n$.

Solution 2. [Y. Zhao] Let the term in the ith row and the jth column of the array be denoted by a_{ij}, and let $S = \{(i,j) : a_{ij} > 0\}$. Suppose that r_i is the sum of the ith row and c_j the sum of the jth column. Then $r_i = c_j$ whenever $(i,j) \in S$. Then we have that

$$\sum\{\frac{a_{ij}}{r_i} : (i,j) \in S\} = \sum\{\frac{a_{ij}}{c_j} : (i,j) \in S\} \ .$$

We evaluate the sums on either side independently.

$$\sum\{\frac{a_{ij}}{r_i} : (i,j) \in S\} = \sum\{\frac{a_{ij}}{r_i} : 1 \leq i \leq m, 1 \leq j \leq n\} = \sum_{i=1}^{m}\frac{1}{r_i}\sum_{j=1}^{n}a_{ij} = \sum_{i=1}^{m}\left(\frac{1}{r_i}\right)r_i = \sum_{i=1}^{m}1 = m \ .$$

$$\sum\{\frac{a_{ij}}{c_j} : (i,j) \in S\} = \sum\{\frac{a_{ij}}{c_j} : 1 \leq i \leq m, 1 \leq j \leq n\} = \sum_{j=1}^{n}\frac{1}{c_j}\sum_{i=1}^{m}a_{ij} = \sum_{j=1}^{n}\left(\frac{1}{c_j}\right)c_j = \sum_{j=1}^{n}1 = n \ .$$

Hence $m = n$.

Comment. The second solution can be made cleaner and more elegant by defining $u_{ij} = a_{ij}/r_i$ for all (i, j). When $a_{ij} = 0$, then $u_{ij} = 0$. When $a_{ij} > 0$, then, by hypothesis, $u_{ij} = a_{ij}/c_j$, a relation that in fact holds for all (i, j). We find that

$$\sum_{j=1}^{n} u_{ij} = 1 \qquad \text{and} \qquad \sum_{i=1}^{n} u_{ij} = 1$$

for $1 \le i \le m$ and $1 \le j \le n$, so that (u_{ij}) is an $m \times n$ array whose row sums and column sums are all equal to 1. Hence

$$m = \sum_{i=1}^{m} \left(\sum_{j=1}^{n} u_{ij} \right) = \sum \{u_{ij} : 1 \le i \le m, 1 \le j \le n\} = \sum_{j=1}^{n} \left(\sum_{i=1}^{m} u_{ij} \right) = n$$

(being the sum of all the entries in the array).

4. Consider a round-robin tournament with $2n+1$ teams, where each team plays each other team exactly once. We say that three teams X, Y and Z, form a *cycle triplet* if X beats Y, Y beats Z, and Z beats X. There are no ties.

(a) Determine the minimum number of cycle triplets possible.

(b) Determine the maximum number of cycle triplets possible.

Solution 1. (a) The minimum is 0, which is achieved by a tournament in which team T_i beats T_j if and only if $i > j$.

(b) Any set of three teams constitutes either a cycle triplet or a "dominated triplet" in which one team beats the other two; let there be c of the former and d of the latter. Then $c + d = \binom{2n+1}{3}$. Suppose that team T_i beats x_i other teams; then it is the winning team in exactly $\binom{x_i}{2}$ dominated triples. Observe that $\sum_{i=1}^{2n+1} x_i = \binom{2n+1}{2}$, the total number of games. Hence

$$d = \sum_{i=1}^{2n+1} \binom{x_i}{2} = \frac{1}{2} \sum_{i=1}^{2n+1} x_i2 - \frac{1}{2} \binom{2n+1}{2} \; .$$

By the Cauchy-Schwarz Inequality, $(2n + 1) \sum_{i=1}^{2n+1} x_i2 \ge (\sum_{i=1}^{2n+1} x_i)2 = n2(2n + 1)2$, whence

$$c = \binom{2n+1}{3} - \sum_{i=1}^{2n+1} \binom{x_i}{2} \le \binom{2n+1}{3} - \frac{n2(2n+1)}{2} + \frac{1}{2}\binom{2n+1}{2} = \frac{n(n+1)(2n+1)}{6} \; .$$

To realize the upper bound, let the teams be $T_1 = T_{2n+2}$, $T_2 = T_{2n+3}$. $\cdots$, $T_i = T_{2n+1+i}$, $\cdots$, $T_{2n+1} = T_{4n+2}$. For each i, let team T_i beat $T_{i+1}, T_{i+2}, \cdots, T_{i+n}$ and lose to $T_{i+n+1}, \cdots, T_{i+2n}$. We need to check that this is a consistent assignment of wins and losses, since the result for each pair of teams is defined twice. This can be seen by noting that $(2n + 1 + i) - (i + j) = 2n + 1 - j \ge n + 1$ for $1 \le j \le n$. The cycle triplets are $(T_i, T_{i+j}, T_{i+j+k})$ where $1 \le j \le n$ and $(2n + 1 + i) - (i + j + k) \le n$, *i.e.*, when $1 \le j \le n$ and $n + 1 - j \le k \le n$. For each i, this counts $1 + 2 + \cdots + n = \frac{1}{2} n(n + 1)$ cycle triplets. When we range over all i, each cycle triplet gets counted three times, so the number of cycle triplets is

$$\frac{2n+1}{3} \left(\frac{n(n+1)}{2} \right) = \frac{n(n+1)(2n+1)}{6} \; .$$

Solution 2. [S. Eastwood] (b) Let t be the number of cycle triplets and u be the number of ordered triplets of teams (X, Y, Z) where X beats Y and Y beats Z. Each cycle triplet generates three ordered triplets while other triplets generate exactly one. The total number of triplets is

$$\binom{2n+1}{3} = \frac{n(4n2 - 1)}{3} \; .$$

The number of triples that are not cycle is

$$\frac{n(4n2 - 1)}{3} - t \; .$$

Hence

$$u = 3t + \left(\frac{n(4n2-1)}{3} - t\right) \Longrightarrow$$

$$t = \frac{3u - n(4n2-1)}{6} = \frac{u-(2n+1)n2}{2} + \frac{n(n+1)(2n+1)}{6}\ .$$

If team Y beats a teams and loses to b teams, then the number of ordered triples with Y as the central element is ab. Since $a+b=2n$, by the Arithmetic-Geometric Means Inequality, we have that $ab \le n2$. Hence $u \le (2n+1)n2$, so that

$$t \le \frac{n(n+1)(2n+1)}{6}\ .$$

The maximum is attainable when $u = (2n+1)n2$, which can occur when we arrange all the teams in a circle with each team beating exactly the n teams in the clockwise direction.

Comment. Interestingly enough, the maximum is $\sum_{i=1}^{n} i2$; is there a nice argument that gives the answer in this form?

5. The vertices of a right triangle ABC inscribed in a circle divide the circumference into three arcs. The right angle is at A, so that the opposite arc BC is a semicircle while arc AB and arc AC are supplementary. To each of the three arcs, we draw a tangent such that its point of tangency is the midpoint of that portion of the tangent intercepted by the extended lines AB and AC. More precisely, the point D on arc BC is the midpoint of the segment joining the points D' and D'' where the tangent at D intersects the extended lines AB and AC. Similarly for E on arc AC and F on arc AB.

Prove that triangle DEF is equilateral.

Solution 1. A prime indicates where a tangent meets AB and a double prime where it meets AC. It is given that $DD' = DD''$, $EE' = EE''$ and $FF' = FF''$. It is required to show that arc EF is a third of the circumference as is arc DBF.

AF is the median to the hypotenuse of right triangle $AF'F''$, so that $FF' = FA$ and therefore

$$\text{arc } AF = 2\angle F''FA = 2(\angle FF'A + \angle FAF') = 4\angle FAF' = 4\angle FAB = 2 \text{ arc } BF\ ,$$

whence arc $FA = (2/3)$ arc BFA. Similarly, arc $AE = (2/3)$ arc AEC. Therefore, arc FE is 2/3 of the semicircle, or 1/3 of the circumference as desired.

As for arc DBF, arc $BD = 2\angle BAD = \angle BAD + \angle BD'D = \angle ADD'' = (1/2)$ arc ACD. But, arc $BF = (1/2)$ arc AF, so arc $DBF = (1/2)$ arc $FAED$. That is, arc DBF is 1/3 the circumference and the proof is complete.

Solution 2. Since $AE'E''$ is a right triangle, $AE = EE' = EE''$ so that $\angle CAE = \angle CE''E$. Also $AD = D'D = DD''$, so that $\angle CDD'' = \angle CAD = \angle CD''D$. As $EADC$ is a concyclic quadrilateral,

$$\begin{aligned}
180^\circ &= \angle EAD + \angle ECD \\
&= \angle DAC + \angle CAE + \angle ECA + \angle ACD \\
&= \angle DAC + \angle CAE + \angle CEE'' + \angle CE''E + \angle CDD'' + \angle CD''D \\
&= \angle DAC + \angle CAE + \angle CAE + \angle CAE + \angle CAD + \angle CAD \\
&= 3(\angle DAC + \angle DAE) = 3(\angle DAE)
\end{aligned}$$

Hence $\angle DFE = \angle DAE = 60^\circ$. Similarly, $\angle DEF = 60^\circ$. It follows that triangle DEF is equilateral.

GRADERS' REPORT

The grading committee consisted of Ed Barbeau, Robert Barrington Leigh, Man-Duen Choi, Felix Recio, Jacob Tsimerman and Ed Wang. Each question was graded by two people, and in the case of the papers in the top quartile, the grade was verified by a third. The top papers were considered by the committee as a whole to agree on the ranking.

Astonishingly and unfortunately, there was an error in the first problem that did not get detected by any member of the committee or the validator. One can only speculate on the group psychological effect at work. The chairman deeply regrets this occurrence. Accordingly, this question was essentially marked out of 5 for a careful explanation leading to a well-posed sum, with 6 being given to one student who produced a particularly fine solution. The remaining questions were marked out of 7. In the ranking, the top papers were considered both with and without question 1. Fortunately, there was no difference with the top three papers, and there was a shuffling of the honorable mentions.

81 students were registered to write the Canadian Mathematical Olympiad. Two failed to submit papers; one student was abroad, and was too engaged to make the attempt; the second wrote the paper on the same day as another examination and chose not to submit it.

The marks award for the several problems are given in the following table:

Marks	**#1**	**#2**	**#3**	**#4**	**#5**
7	0	31	5	2	8
6	1	11	1	0	0
5	8	5	1	0	1
4	3	1	2	2	1
3	13	3	5	13	1
2	16	12	9	41	2
1	21	4	15	1	3
0	5	8	27	7	25
–	12	4	14	13	38

Problem 1. Students could gain 2 marks for obtaining, with justification, the correct number of ways of distributing 1 and 4 candies. For 5 points, students had to give a correct sum, either in closed form with the limits of summation clearly delineated or in extensive form, along with a description of the reasoning. Many students gave an answer without anything in the way of justification, and quite a few did not keep the indices straight. This question points up the need for competitors to justify the expressions they put down and to give as much of the solution as they can, even if they cannot, or think that they cannot, complete the problem. Credit was given for recognizing the relation

$$f(2006, k) = f(2005, k) + f(2005, k-1) + f(2005, k-2)$$

which was an important ingredient in the intended problem, and a tool for simplifying the expression in this one.

2. For most students, this was the most straightforward question on the paper, and the method of choice was analytic geometry. However, many solutions were hardly fluent in this technique, and some had solutions with excessive use of subscripts. Many students were not careful about indicating which part of the straight line was actually the locus.

3. While many students had a rough idea of what was at stake, they quickly got bogged down. Many started out with a positive entry in, say, a row, then looked at the columns involved, and then the rows involved with these columns, and so on, entering into a spiral of consequences that they could not control. The way around this is to try to define at the beginning the end result of this spiralling situation, either by considering the set of all rows and columns that share a particular sum, or, in the case of one student, considering a minimum block of entries of the array which contained along with each nonzero entry, all the nonzero entries belong to the same row or column. This then set up the basis for an induction argument.

Several students began their solutions with faulty assumptions, for example that one could rearrange rows and columns to achieve a situation in which, in the case $m < n$, there was a $m \times m$ submatrix with a diagonal of nonzero entries with the remaining entries zero. However, the very elegant argument (Solution 2) by Yufei Zhao cuts through any need for complicated reasoning and possible confusion, and should be studied by students for possible use elsewhere.

4. Many students got the easy (a) part of the problem, for which 2 points were awarded, but very few proceeded any further. However, a few solutions to (b) were quite elegant and succeeded by making a count of cycle and non-cycle triplets as in the official solutions. Some tried an induction argument, but got fouled up when they added an additional pair of teams; no one succeeded with this approach.

5. This problem was better done than anticipated, with many students relating angles in a circle subtended by the same chord, identifying the exterior angle of a triangle with the sum of the interior and opposite triangles, and equating the angle between tangent and chord with the angle subtended by the chord. Some students overdid the symmetry. While the argument showing that angle DEF was equal to 60 degrees was analogous to that for angle DFE, angle EDF needed a separate treatment. A few students were mislead by the diagram and took BC to be parallel to $D'D''$ or assumed some other fact that depended on triangle ABC being isosceles.