Among probability measures on $\mathbb{R}^n$ which take their values in a set of unit diameter, we show the variance around the mean is maximized precisely by those measures which assign mass $1/(n+1)$ to each vertex of a (unit-diameter) regular simplex (i.e. an equilateral triangle in two dimensions and regular tetrahedron in three). This provides sharp generalizations of variance bounds by Popoviciu (1935) and Bhatia-Davis (2000) from $n = 1$ to higher dimensions.

Based on https://arxiv.org/abs/1907.13593 and work in progress with Tongseok Lim.