Consider an open Riemann surface $\Sigma$ of genus $g > 0$ with $n > 1$ borders, each one homeomorphic to the unit circle. The surface $\Sigma$ can be described as a compact Riemann surface $R$ of the same genus $g$, from which $n$ simply-connected domains $\Omega_1, \ldots, \Omega_n$, removed; that is, $\Sigma = R \setminus \cup \text{cl}(\Omega_k)$. Fix conformal maps $f_k$ from the unit disc $\mathbb{D}$ onto $\Omega_k, k = 1, \ldots, n$. We may assume each $f_k$ has a quasiconformal extension to an open neighbourhood of $\mathbb{D}$. Let $f = (f_1, \ldots, f_n)$.

I will define the Grunsky operator $Gr_f$ corresponding to $f$ (equivalently to $\Sigma$) on some Dirichlet spaces when all the boundary curves are quasicircles in $R$. I will show that the norm of the Grunsky operator is less than or equal to one. This is a generalization of the classical Grunsky inequalities from the planar case to bordered Riemann surfaces described above.

Joint work with E. Schippers and W. Staubach.