Some questions on finite groups motivated by the complexity of QCSP

Let $G$ be a finite group, and consider the following decision problem: given a quantified system of equations such as

$$\exists x \forall y \exists z \ x^2 y^3 = z^4 x z \land y^2 z^5 = x \land \cdots,$$

is it valid? Recent developments in the algebraic theory of the computational complexity of quantified constraint satisfaction problems (QCSP) lead us to study the nature of the surjective operations compatible with the group operation, i.e. the group epimorphisms of the form $f : G^n \to G$. We show, amongst other results, that if $G$ is a quasi-simple group, then all these operations are essentially unary, whence the associated QCSP is PSPACE-complete.

(Work in progress with Barnaby Martin, University of Durham, UK)