Finite planes, ZZ-topes and Fibonacci Numbers.

Let $F_p$ denote the finite field of order $p$ and $F$ its algebraic closure. Classifying the $F$-representations of $\mathbb{Z}/p\mathbb{Z} \times \mathbb{Z}/p\mathbb{Z}$ leads to a simply stated geometric problem involving the $F_p$-planes contained in $F$. Solving this leads in turn to an infinite sequence of polynomials $f_{p,r}(t) \in F[t]$ for $r = 1, 2, 3, \ldots$.

It turns out that we may describe these polynomials uniformly in terms of $p$. This description allows us to generalize to any integer value of $p$. Taking $p = 1$ we recover a classical family of orthogonal polynomials the Morgan-Voyce polynomials which originally arose in a study of electrical resistance in 1959. The Morgan-Voyce polynomials are known to have many connections with the Fibonacci sequence. Our approach yields a new description of the Morgan-Voyce polynomials in terms of an infinite sequence of binary vectors.

Using certain initial segments of this sequence as the vertices of a polytope, we recover the zigzag order polytopes. These polytopes were considered by Stanley and shown to have strong connections with certain elements of the permutation group and with zigzag (or fence) posets. If time permits we will also illustrate a number of other surprising properties of this sequence of binary vectors.

This is joint work with H.E.A. Campbell (UNB).