Inverse Problems in Mathematical Physics
Problèmes inverses en physique mathématique
(Org: **Peter Gibson** and/et **Yue Zhao** (York University))

LUZ ANGÉLICA CAUDILLO-MATA, University of British Columbia
Multiscale Modeling of Geophysical Electromagnetic Fields in Highly Heterogeneous Media

In this work, we present two multiscale methods with and without oversampling. These methods simulate quasi-static electromagnetic responses in the frequency domain. They lead to a significant reduction in the size of the final system of equations and the computational time. For showing the performance of these methods, we construct a synthetic 3D geophysical setting of a mineral deposit from inversion results of field measurements over the Canadian Lalor mine. The setting includes highly heterogeneous conductive media, features varying at several spatial scales and it is discretized using OcTree meshes.

This work is done in collaboration with Prof. Eldad Haber (UBC), Dr. Christoph Schwarzbach (Computational Geosciences, Inc), and Dr. Wenke Wilhelms (UBC).

PETER GIBSON, York University
Acoustic imaging of layered media

We present a recent solution to the inverse medium problem in one dimension. The echoes-to-impedance transform is a nonlinear transform designed for acoustic imaging of layered media. The transform converts time domain digital reflection data directly into impedance as a function of spatial location, using minimal prior information about the source wavelet and no pre-preprocessing. It is simple, fast, and, according to numerical experiments, highly accurate. More than this, physical structure is superresolved at a finer scale than that of the source wavelet. The derivation of the echoes-to-impedance transform stems from a recently developed numerical method for wave propagation in one dimension in conjunction with the theory of orthogonal polynomials on the unit circle.

ISAAC HARRIS, Purdue University
Transmission Eigenvalues for a Conductive Boundary

In this talk, we will investigate the inverse acoustic scattering problem associated with an inhomogeneous media with a conductive boundary. We consider the corresponding interior transmission eigenvalue problem. This is a new class of eigenvalue problem that is not elliptic, not self-adjoint, and non-linear, which gives the possibility of complex eigenvalues. We investigate the convergence of the eigenvalues as the conductivity parameter tends to zero as well as prove existence and discreteness for the case of an absorbing media.

MICHAEL HASLAM, York University
A High Order Method for Electromagnetic Scattering from Penetrable Rough Surfaces

The problem of evaluating the electromagnetic response of a periodic surface to an incident plane wave is of great importance in science and engineering. Applications of the theory exist in several fields of study including solar energy research, optical instrument design, and remote sensing. A fast solver for the forward problem, as we describe here, often forms the basis of an iterative solver for the inverse problem. We discuss the extension of our previous methods (J. Opt. Soc. Am. A 26(3): 658–668, 2009; Waves in Random and Complex Media 20(4): 530–550, 2010) to treat the problem of scattering from penetrable surfaces with a complex refractive index. The generalization of our methods is not straight-forward, and involves the careful treatment of certain hyper-singular operators which arise in the formulation of the problem in terms of surface integral equations. We demonstrate the rapid convergence of our methods for classically difficult cases in the optical sciences.

RU-YU LAI, University of Minnesota
Global uniqueness for the semilinear fractional Schrödinger equation

In this talk, I will briefly address an emergent inverse problem for nonlocal operators which can be regarded as a nonlocal analogue version of the well-known Calderón problem. Several unique features in this type of problems will be discussed. Moreover, I will present a uniqueness result in inverse problem for a fractional semilinear Schrödinger equation.

MICHAEL LAMOUREUX, University of Calgary
Inverse theory in imaging

We discuss recent developments in the application of mathematical inverse theory to the problem of seismic imaging for exploration of commercial hydrocarbon resources. Full waveform inversion is a particularly challenging area, both theoretically and computationally, given the limitation of real acquired data in the fields and the large scale numerical computations required.

WENYUAN LIAO, University of Calgary
A numerical method for earth mover's distance calculation and its application in full waveform inversion

Conventional full waveform inversion (FWI) using least square distance (LSD) suffers from local minima. Recently, earth mover's distance (EMD) has been introduced to FWI to compute the misfit between two seismograms. Instead of comparisons bin by bin, EMD allows to compare signal intensities across different coordinates. This measure has great potential to account for time and space shifts of events within seismograms. However, there are two main challenges: (1) the compared signals need to satisfy non-negativity and mass conservation assumptions; (2) the computation of EMD between two seismograms is expensive. In this work, a strategy is used to satisfy the two assumptions via decomposition and recombination of original seismic data. The computation of EMD based on dynamic formulation is formulated as a convex optimization problem, which is solved by a primal-dual hybrid gradient method with linesearch on GPU device. The new method is efficient and also easy to implement. A 1D time-shift signals case study has indicated that EMD is more effective in capturing time shift. Two applications to synthetic data using transmissive and reflective recording geometries have demonstrated the effectiveness of EMD in mitigating cycle-skipping issues. The new method has been applied to SEG 2014 benchmark data. The result demonstrated that EMD can mitigate local minima and provide reliable velocity estimations without using low frequency information.

BAS PETERS, University of British Columbia
Intersections and sums of sets for the regularization of inverse problems

We present new algorithms to compute projections onto the intersection of constraint sets. We focus on problems with multiple sets for which there is no simple and closed-form projection. Different from more classical methods such as Dykstra's algorithm, we do not need other algorithms to solve sub-problems.

Our algorithms are based on the alternating direction method of multipliers and apply to models/images/video on small 2D and large 3D grids because we exploit computational similarity between constraint sets, coarse and fine-grained parallelism, and we also present a multilevel accelerated version.

To obtain more flexible descriptions of prior knowledge, we introduce a formulation that allows constraint sets to be the sum of intersections of sets, which is essentially an extension of a Minkowski set. This formulation builds on the success of additive image descriptions that are usually based on penalty methods, such as cartoon-texture decomposition and robust principal component analysis.

We show applications where we use multiple constraint sets to regularize partial-differential-equation based parameter estimation problems such as seismic waveform inversion, as well as various image and video processing and segmentation tasks.

ALEXANDRE TIMONOV, University of South Carolina Upstate
On the large time behavior of the solution to the regularized weighted mean curvature flow problem

An initial boundary value problem for the weighted mean curvature flow equation in a smooth bounded domain is considered. Under an assumption that the boundary has the positive mean curvature, and a weight function is positive, the problem is regularized to eliminate the singularity and degeneracy of the differential operator. The Rothe's method is applied to reduce the resulting quasi-linear parabolic problem to a family of the linear elliptic problems. Each of these problems in every temporal layer is numerically solved using the regularized successive approximations. In view of applications to Current Density Impedance Imaging, the large time behavior of the numerical solution to the regularized weighted mean curvature flow problem is investigated. In the numerical experiments it is shown that for a sufficiently large time the regularized solution is approaching the equilibrium solution, i.e., the function of the Dirichlet weighted least gradient. However, the numerical convergence is no more than linear. Therefore, further studies, analytical and numerical, will be done to establish the global convergence result for the original problem and to improve the rate of convergence. The work is supported in part by the NSF grant DMS-181882.

JIAN ZHAI, University of Washington
A semiclassical inverse spectral problem for elastic surface-wave tomography

More than a decade ago, Campillo and his collaborators discovered that cross correlation of ambient noises yields Green's function for surface waves. Since then, seismologists began to use these noises to do a "passive imaging" of the geological structure of the Earth. We use a semiclassical framework to describe surfaces in an elastic half space that is stratified near the boundary at some scale comparable to the wave length. The analysis is based on the work of Colin de Verdiere on acoustic surface waves. A one-dimensional ODE operator, parametrized in phase space, have its eigenvalues governing the geometrical behavior of surface waves. Recovery of material parameters from dispersion relations of surface waves can be formulated as an inverse spectral problem for the ODE operator. Under certain generic conditions, we give a reconstruction scheme for the S-wave speed with multiple wells.

YUE ZHAO, York University
Inverse source problems in electrodynamics

This talk concerns inverse source problems for the time-dependent Maxwell equations. The electric current density is assumed to be the product of a spatial function and a temporal function. We prove uniqueness and stability in determining the spatial or temporal function from the electric field, which is measured on a sphere or at a point over a finite time interval.