While Euler and others of the mid-18th century had methods for assigning a value to some divergent series, the broad adoption of the Cauchy definition for the sum of a series made such objects problematic. However, by the mid-19th century, renewed interest in these series, as a result of their usefulness in physics, appeared at the hands of Stokes and Poincaré. In this talk I will look at some of the objections to divergent series as well as the reasons why they continued to appear and were clearly useful in some applications. Associating a divergent series with a convergent integral, finding a divergent series solution to a differential equation, or using a divergent series for asymptotic approximations made clear that these series were useful but also that a more coherent mathematical theory was needed. The consideration of these issues led to the development of new techniques near the turn of the 20th century.