We consider the problem of finite generation of the Cox ring for blowups of weighted projective planes at a generic point of the torus.

By a result of Cutkosky, finite generation for these spaces is equivalent to the existence of two different curves in the varieties: (1) a "negative curve" different from the exceptional curve and (2) another curve disjoint from the previous one. Toric geometry gives us control over condition (1). By work of Kurano and Nishida, the second condition can be tested by considering the problem in positive characteristic for all primes big enough.

In this talk we'll describe this reduction method to prime characteristic as well as the obstructions to lift the solutions back to characteristic zero. We'll conclude by presenting our results and the open problems in the area. This is joint work with J. L. González and K. Karu.