We present an extension to a method for constructing solvable diffusions. This gives rise to new solvable models that are divided into two main classes; the first is specified by having a linear drift with various resulting nonlinear diffusion coefficient functions, while the second class allows for several specifications of (generally) nonlinear diffusion coefficient functions with resulting nonlinear drift function. The first class of models is useful for pricing equity and foreign exchange (FX) options in finance, while the second class contains new models that are mean-reverting and are applicable to pricing interest-rate and other path-dependent derivatives such as volatility index (VIX) options. As specific examples of the first class, we present explicit results for two new families of solvable models. For the second class, we give examples of new solvable nonlinear mean-reverting processes and derive closed-form integral formulas for conditional expectations of certain functionals. In particular, we derive a new closed-form analytical formula for the Laplace transform with respect to the strike price of a standard call VIX option. We then succeed in Laplace inverting the expression to obtain exact VIX call option prices with realistic implied volatilities with respect to strike and maturity. Lastly, we accurately calibrate another of our new models to USD/EUR FX option market data exhibiting pronounced implied volatility smiles across several strikes and maturities.