A plane curve is nothing more than a polynomial in two variables. Viewed over the complex numbers, this describes a surface, possibly with singularities. When no such singularities are present, the curve is “smooth” and describes a special kind of Riemann surface. If one has a *loop* of such smooth plane curves, there is an associated element of the mapping class group; the collection of all such mapping classes is called the “monodromy group of plane curves”. This talk concerns the basic question: which mapping classes can arise this way? Is the subgroup finite index? I will describe my recent work which gives an essentially complete answer to these questions, and more. The methods involve a combination of geometric group theory and tropical algebraic geometry. Along the way, we will see connections between this problem and some interesting new subgroups of the mapping class group for which many basic questions are not yet answered.