A new infinite family of hemisystems of the Hermitian surface

This is a talk about tight sets and $m$-ovoids of the classical polar spaces (in particular, quadrics). Tight sets and $m$-ovoids are important substructures of the classical polar spaces, which are not only interesting in their own right, but also can give rise to many other geometric/combinatorial objects, such as translation planes, strongly regular graphs, two-weight codes. We will talk about a recent construction of hemisystems of the Hermitian surface $H(3, q^2)$ in the case where $q \equiv 3 \pmod{4}$, or equivalently, a $\frac{(q+1)}{2}$-ovoid of $Q^-(5, q)$, the elliptic quadric in $PG(5, q)$. The construction uses cyclotomic classes of finite fields, and it depends on rather complicated computations involving Gauss sums. The talk is based on joint work with John Bamberg, Melissa Lee, and Koji Momihara.