Quantum walks in association schemes

The continuous-time quantum walk on a graph $X$ is given by the unitary operator $e^{-itA}$, where $A$ is the adjacency matrix of $X$. The graph $X$ admits fractional revival from $u$ to $v$ at time $\tau$ if

$$e^{-i\tau A} = \alpha e_u + \beta e_v,$$

for some $\alpha, \beta \in \mathbb{C}$. Here $e_u$ and $e_v$ denote the characteristic vectors of vertices $u$ and $v$, respectively.

Perfect state transfer from $u$ to $v$ and periodicity at $u$ are two special cases of fractional revival with $\alpha = 0$ and $\beta = 0$, respectively. These two properties have been extensively studied but not so much for fractional revival when both $\alpha$ and $\beta$ are nonzero.

Instantaneous uniform mixing is another interesting phenomenon of the continuous-time quantum walk on a graph. This happens when $\sqrt{n}e^{-i\tau A}$ is a complex Hadamard matrix.

In this talk, we look for graphs in association schemes that satisfy one or more of these phenomena.