In this talk, I show two lower bounds of \( h_2 \)-numbers of simplicial complexes in terms of the minimum number of generators of their fundamental groups. I will also explain a nice relation between the polyhedral Morse theory and graded Betti numbers, which is used to relate numbers of generators of fundamental groups and \( h_2 \)-numbers.

This is a joint work with Isabella Novik.