A discrete group is said to be C*-simple if its reduced C*-algebra is simple. It is not difficult to see that a group with this property does not have any non-trivial normal amenable subgroups, however it was an open question for many years to determine whether the converse holds. Recent examples constructed by Le Boudec show that the answer to this question is negative, but raise the question of whether there is an intrinsic algebraic characterization of C*-simplicity. In this talk I will discuss recent work providing such a characterization.