Utility maximization in a regime switching model with convex portfolio constraints and margin requirements

We study a problem of stochastic control in mathematical finance, with the goal of maximizing expected utility of investment and consumption over a finite trading horizon. The asset prices are modeled by Itô processes, for which the market parameters are subject to regime switching in the sense of being adapted to the joint filtration of the driving Brownian motion and a finite-state Markov chain which models “regime states” of the market. The vector of portfolios is constrained to a specified closed and convex set, and margin payments are levied on the investor, resulting in a wealth equation which is nonlinear in the portfolio. We proceed by the method of conjugate duality to construct a dual optimization problem together with optimality relations between putative solutions of the given (i.e. “primal”) optimization problem and the dual optimization problem. These optimality relations are then used to address the specific cases of power-type and logarithmic utility functions, with convex cone portfolio constraints, and a higher rate of interest for borrowing than for lending. We get completely explicit optimal portfolios and characterize the optimal consumption rate as the solution of a backward stochastic differential equation (BSDE) “driven” by the canonical martingales of the regime-state Markov chain. For the power utility function this is a rather unconventional BSDE, to which standard existence results do not apply, and accordingly we establish existence and uniqueness of solutions for this BSDE.