Majorization is a concept that emerges from the properties of stochastic matrices. The theory of majorization has been identified as an important tool in quantum information since its application to the theory of entanglement by Nielsen. Later works have identified other areas of quantum information where it plays a role, including thermodynamics. In this talk, we describe the connection between majorization theory and thermodynamics, with a summary of our recent results in this area. These include necessary and sufficient conditions for transitions between thermodynamic states of quantum systems, under various conditions (with or without catalysts, costing or yielding work, small or large systems). Notably, our results imply the insufficiency of the traditional formulation of the Second Law to decide the feasibility of state transitions. Based on the work in http://arxiv.org/abs/1309.6586